# AUTONOMOUS TWO-DIMENSIONAL FIRST ORDER RECURSIVE DIGITAL FILTERS WITH OVERFLOWOSCILLATIONS NONLINEARITY WITHOUT QUANTIZATION

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*Abstract* - **Nonlinear properties of the two -dimensional first order recursive digital filter with saturation nonlinearity permit to use it for generation of 2-D signals and images. Areas, in which concrete types of two-dimensional limit cycles in coefficients space of the filter can exist, are received. Bifurcation diagram of two-dimensional first order recursive digital filter is analyzed.**

*Index terms* – **Two-dimensional first-order recursive digital filter, saturation nonlinearity, overflow effects, limit cycles.**

## I. Introduction

Nonlinear problems in digital filters can be divided into three main classes [1]:

1) research of processes in the systems with the linear adder and quantization;
2) research of processes in the systems with the nonlinear adder and quantization;
3) research of processes in the systems with the nonlinear adder without quantization.

The problems connected with the research of the first class for two-dimensional systems are in an initial stage, though similar one-dimensional problems are well studied. The majority of works is dedicated to the analysis of one-dimensional nonlinear systems with the use of statistical approach [2]. With its help it is possible to determine an average level of noise of quantization on an output of nonlinear system and its capacity. The disadvantage of the statistical approach is the certain rigidity of initial requirements for its application that is not always carried out in practice.

Systems with the nonlinear adder and quantization typically have two-dimensional limit cycles on their output. Considering this, the research of the given class of systems is conducted using the determined approach, thus allowing us to find areas of existence and parameters of two-dimensional limit cycles. The definitions of two-dimensional limit cycles are given in [3]. Necessary and sufficient conditions for appearance of confluent limit cycles in first order filters with three quantizers and necessary conditions for appearance of diagonal limit cycles in the special case of first order equation are found in [4].

Theorems, reflecting some general properties of two-dimensional first order digital filters with two nonzero coefficients and with one quantizer, working on a principle of a roundoff are proved in [5]. For a case of three-level quantization, areas of existence of two-dimensional limit cycles in space of coefficients of the filter are found in [6]. Also such parameters of cycles as an amplitude and a period are determined there.

In the works dedicated to research of systems with the nonlinear adder without quantization, conditions of stability in state-space are considered. Sufficient conditions of stability of Fornasini-Marchesini state-space model are found in [7]. Sufficient conditions of asymptotic stability of the nonlinear two-dimensional filter are also considered there. Lyapunov second method for a determination of sufficient conditions of global asymptotic stability of two-dimensional filter state-space model is used in [8]. Observance of conditions of stability guarantees the absence of limit cycles on an output of the filter. During solution of the given class of problems not much attention is usually paid to areas of existence of two-dimensional limit cycles and their analysis.

Practical interest represents detection of various cycles, which can exist in autonomous system with the nonlinear adder without quantization and areas of their existence in the coefficients space.

The problems connected with studying conditions of existence of limit cycles with the different periods as a result of the adders nonlinearity and with an estimation of their amplitude taking into account the effects of quantization, can be solved with the help of the determined approach [9, 10]. The essence of the approach consists in splitting a range of definition of function of nonlinearity into zones with various values. Then, consistently analyzing possible transitions of system on these zones, restrictions on the parameters of the system corresponding to certain movements are determined. As a result all space of parameters of system can be split into areas with various types of movements. In addition it is necessary to consider function of nonlinearity ofthe adder in an exp licit form.

## II. Model Formulations

Let's consider the autonomous two-dimensional first order recursive digital filter with the account of only nonlinearity of the adder. Such model is correct in case when the number of levels of quantization of arithmetic operations is sufficiently large, i.e. effects of quantization do not affect work of the filter.

In this work the two-dimensional first order recursive digital filter, described by the following nonlinear difference equation is considered:

$$\begin{aligned} y(n, m) = f(b_{10}y(n-1, m) + \\ + b_{01}y(n,m-1) + \\ + b_{11}y(n-1,m-1)), \end{aligned} \tag{1}$$

where $b_{10}$, $b_{01}$, $b_{11}$ - coefficients of the filter, and function $f(\cdot)$ – is the characteristic of the adder. In practice commonly the following characteristic of the adder with saturation (Fig. 1) is used

$$f(\varphi) = \begin{cases} \varphi, & for\ |\varphi| < 1, \\ sign(\varphi), & for\ |\varphi| \ge 1. \end{cases} \tag{2}$$

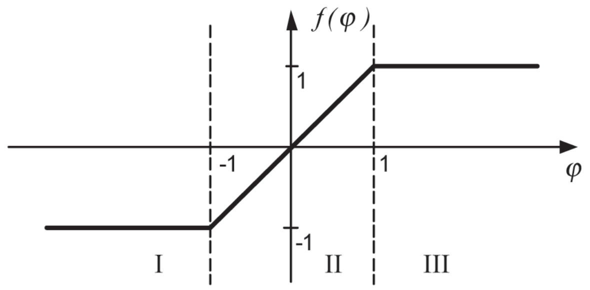


Fig. 1. Saturation nonlinearity

Initial conditions are set as follows:

$$\begin{cases} y(n,-1) = y(-1,m) = 1, & for\ n = m = -1, \\ y(-1,m) = y(n,-1) = 0, & for\ m > -1, n > -1. \end{cases} \tag{3}$$

## III. Main results

The area of a linear mode of the filter is defined by the inequality:

$$|b_{10}| + |b_{01}| + |b_{11}| < 1.$$

According to the method of research, the characteristic of the adder is divided into three zones (Fig. 1).

Let's consider behavior of system (1) for $n \to \infty$. Taking the initial conditions (3) into account, we have:

$$y(0,0) = f(b_{11}y(-1,-1)) = f(b_{11}),$$
$$y(1,0) = f(b_{10}y(0,0)) = f(b_{10}b_{11}),$$
$$y(2,0) = f(b_{10}y(1,0)) = f(b^{2}{}_{10}b_{11}),$$
$$\ldots$$
$$y(n,0) = f(b_{10}y(n-1,0)) = f(b^{n}{}_{10}b_{11}).$$

Thus, for a case $|b_{10}| > 1$, beginning with some $n_1 > N$, the mode $y(n,0) = 1$ is established, and in case that $b_{10} < -1$ sequence is alternating in sign. From Fig. 1 it is visible, that transition from a zone II of function of nonlinearity is carried out when the module of function argument (2) achieves 1. The value of transient time $N$ can be found using the following expression

$$|b^{N}{}_{10}b_{11}| = 1$$
$$N = \mathrm{mod}(1 - \log_{|b_{10}|}|b_{11}|).$$

Applying the formula of transformation of logarithms we derive the final formula of calculation of the transient time $y(n,0)$ receive

$$N = \mathrm{mod}(1 - \frac{\ln|b_{11}|}{\ln|b_{11}|}).$$

If $|b_{10}| < 1$, then $y(n,0) \to 0$ and periodic movements are impossible. Let's consider the following iteration on an axis $m$

$$\begin{aligned} y(n,1) = f(b_{10}y(n-1,1) + b_{01}y(n,0) + \\ b_{11}y(n-1,0)) \end{aligned}.$$

For prescribed value $b_{10} > 1$

$$y(n,1) = f(b_{10}y(n-1,1) \pm (b_{01} + b_{11}));$$

for the case $b_{10} < -1$

$$y(n,1) = f(b_{10}y(n-1,1) \pm (b_{01} - b_{11})),$$

i.e. since the $N_1 > N$, the sequence $y(n,1)$ becomes periodic with the period 1 or 2 and amplitude $|y(n,1)|=1$. Arguing in the same way for a case $m = 2,3,...,\infty$, we have received, that for given $m = const$ sequence $y(n,m)$ is periodic with the period which is not exceeding 2, and amplitude $|y(n,m)|=1$. Character of oscillations is similar for $m \to \infty$, $n = const$. Thus, existence of diagonal limit cycles with the various periods is possible in the system. It is determined, that the basic types of the periods are (1,1), (1,2), (2,1), (2,2). Thus, the amplitude of cycles is equal to 1, and the values $y(n,m)$ belonging to a cycle, are in zones I and-or III (according to Fig. 1).

At Fig. 2 the bifurcation diagram is given and possible kinds of 2-D limit cycles on a plane ($b_{10}$, $b_{01}$) for $-1 < b_{11} < 1$ are represented. It is necessary to note, that values of sequence $y(n,m)$ in a cycle of period (1,1) for the case $-1 < b_{11} < 0$ belong to zone I, and for $0 < b_{11} < 1$ - to a zone III.

Further, row and column limit cycles of the various periods can exist in some zones. For a case when the coefficient $b_{11}$ changes in limits from -1 up to 0, column cycles appear in areas with the periods (1,1) along an axis $m$ and (2,2) along an axis $m$. If the coefficient $b_{11}$ belongs to an interval from 0 to 1, column cycles appear in areas with the periods (1,2) along an axis $m$ and (2,1) along an axis $m$.

In the cube in coefficients space $|b_{10}| < 1$, $|b_{01}| < 1$, $|b_{11}| < 1$ there can exist two kinds of movements essentially differing in character. In the tetrahedron of stability, movements arise in a zone II, then gradually fade and converge to zero. In the rest of space of a cube outside a tetrahedron of stability can exist some periodic movements with sector character, and their kind is defined by areas, adjacent with given. The example of such sector periodic movements is represented at Fig. 3. There black color designates values $y(n,m) = -1$, (zone I), white color – values $y(n,m) = 1$ (zone III), and gradations of grey correspond to values $y(n,m) \in (-1,1)$, belonging to a zone II.

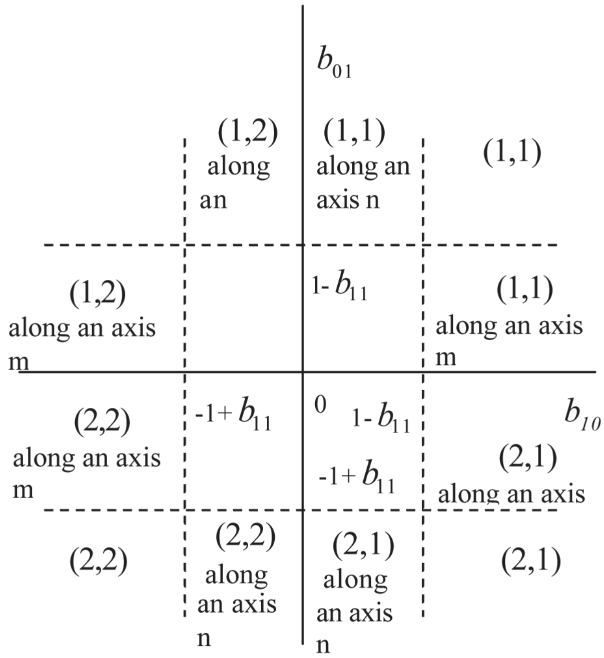


Fig. 2. Bifurcation diagram for a case $-1 < b_{11} < 1$

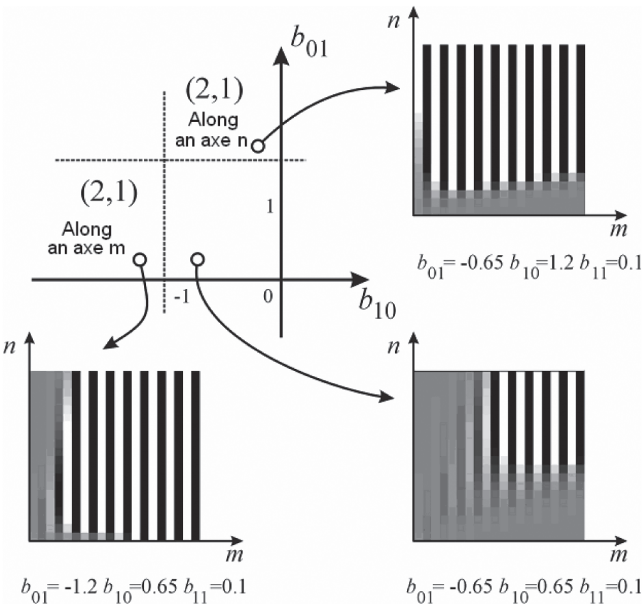


Fig. 3. An example of movements in areas for $0 < b_{11} < 1$

Existence of row limit cycles is probable in areas with the periods (1,1) and (2,2) along an axis $m$ for

$-1 < b_{11} < 0$ and in areas with the periods (1,2) and (2,1) along an axis $n$ for $0 < b_{11} < 1$.

## IV. Conclusions

The opportunity of existence of various types of limit cycles in the two -dimensional first order recursive digital filter with saturation nonlinearity is investigated. Areas, in which concrete types of 2-D limit cycles can exist, are received in coefficients space of the filter. Results are shown in the convenient graphic form. Knowledge of 2-D limit cycles peculiarities can help to avoid some undesirable effects connected with it. Nonlinear properties of the two-dimensional first order recursive digital filter with saturation nonlinearity permit to use it for generation and processing of 2-D signals and images [11].

## V. Acknowledgments

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