SINGLE OP-AMP HIGH-ORDER CT ARC-FILTERS FOR OPERATION AT VERY HIGH INTERFERENCE LEVEL IN STOPBAND

G. N. Slavsky, *St.Petersburg State Polytechnical University*

Abstract – **A new approach to the long –appeared problem is considered. The single Op-Amp high-order ARC filters use 2 to 8 times less number of Op-Amps compared to other type ARC filters of the same order. This design provides lower power consumption (important for portable devices, geophysical "in-the-field" or underwater instruments), as well as simpler network and technology. Fundamentally important is the capability of these filters to operate at very high interference level in stop-band, what may be crucial in certain problems of instrumentation. Filters represented below are considered from this point of view. The procedure of the design of certain filters is briefly presented and illustrated with examples.**

Index Terms – **High-Order, Single-Op-Amp, ARC, High interference level.**

I. INTRODUCTION

The history of high-order ARC-filters on one OA began in the nineteen – sixties and seventies with the appearance of the works by Aggerwal [1] and others, showing the possibility of realization, in principle, of a *n*-th order transfer function via an ARC-circuit based on a single ideal OA. There appeared numerous papers on this subject (list of numerous references is presented in [2]). Unfortunately, filters of this type were not widely used, and interest in them subsided, which, in the opinion of the author, was a mistake.

The author presents certain results of his work in this area, in respect to capacity of the filters under consideration to endure very high interference.

The structure schematic of Single Op-Amp filter is presented in Fig. 1., where *n*-order ladder RC-circuit is connected up to an amplifier *K* equipped with multiloop feedback B_k (note that the amplifier gain is close to 1).

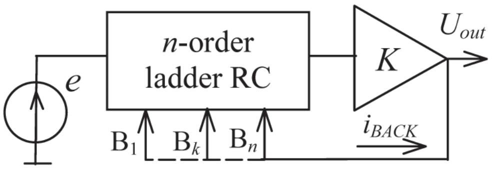

Fig. 1. Structure schematic of high-order ARC filter

The transfer function Uout/e of the schematic may be obtained with a routine procedure valid for ARC-circuits with feedback (see, for example, [3],[4]). The transfer coefficients of ladder RC-circuit, U/e (when Uout=0) and U/Uout (when e=0), required for this operation, may be derived, in particular, by the methodology in [5], [6].

The design procedure, as usual, consists in calculation of the schematic parameters according to the given transfer function. This is illustrated with examples.

The sensitivity (to R & C) and noise of these filters are calculated with routine procedure. These values lie in the usual range. The sensitivity to Op-Amp is higher than that of cascade filters. There is the design procedure to eliminate or decrease this dependence (see below). The dynamic range is usually higher compared to other types of ARC-filters.

II. OPERATING AT HIGH INTERFERENCE IN STOPBAND

A. Unsuitable approach to the problem

The most of ARC-filters of other types, especially based upon the simulation of LC-circuits (Fig.2), are not capable to operate at very high input voltage level because of high "overvoltage" U_{inner} at inner stages.

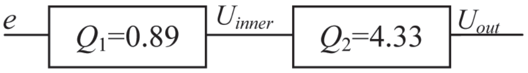

Fig. 2. Structure schematic of two-stage 4-th order LPF

This is illustrated by Fig. 3, where the two realizations of 4-th order Tshebyshev filter (1.74 dB ripple) are compared. The curve 1, concerning to LC-simulating filter in Fig.2, depicts the relative voltage U_{inner}/e on the output of the first stage when voltage e grows as $(f/f_N)^4$ (curve 3; f_N is the cutoff frequency). The alternative filter is free of the overvoltage. The curve 2 simultaneously depicts the output of both the filters in the same conditions (when input voltage is reasonable low).

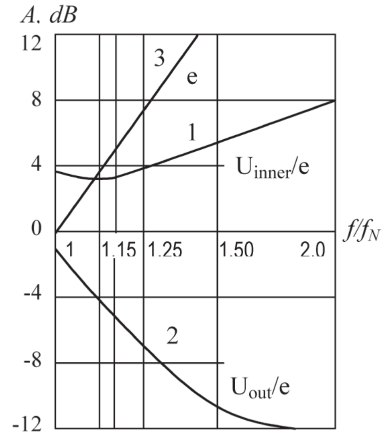

Fig. 3. Dependencies of U_{inner} and U_{out} on f/f_N for 4-th-order LPF

It is evident from the curve 2 that if the input rated signal, for example, is equal to 1 V; signal/interference ratio is required >10 dB, then the input interference level in stopband at $f>2f_N$ may be more than 19 V.

In schematic terms, this phenomenon is caused by the availability of the linear ladder RC-circuit connected in front of the amplifier. In the case of special choose of RC-elements and proper schematic, the interference voltage, endured by the filter, may be very high (up to 100 V and more). For this purpose, we have to design schematic so as the "back" current i_{BACK} in Fig.1, caused by high interference voltage e_{INT} , would be negligible from the standpoint of amplifier operating.

III. DESIGN OF 4-TH-ORDER POLYNOMIAL LPF FOR OPERATING AT HIGH INTERFERENCE

A. Procedure of the filter design

The given transfer function takes the form

$$T(s) = K/(a_4 s^4 + \ldots a_k s^k + \ldots a_1 s + 1), \qquad (1)$$

where: $s = i\omega/\omega_N$, $\omega = 2\pi f$, f is frequency,
$\omega_N = 2\pi f_N$, f_N, is cutoff frequency,
i is the square root of -1.

We use the methodology from [7] for the structure schematic in Fig.1 and the detailed schematic in Fig.4.

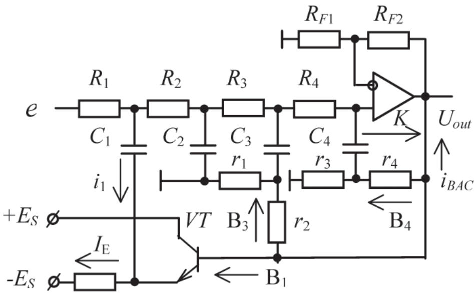

Fig. 4. The 4-th order Polynomial LPF

We employ the regular RC-circuit with parameters:

$$R_1/R_2 = R_2/R_3 = R_3/R_4 = m,\ \ C_1/C_2 = C_2/C_3 = C_3/C_4 = q, \qquad (2)$$

where:

$$R_4 = R,\ \ C_4 = C,\ \ RC = 1/\omega_0,\ \ \omega_0 = g\omega_N, \qquad (3)$$

g is the coefficient of the frequency shift.

For LPF under consideration: values m=0.5 and q=2 are recommended, and all the next formulae are valid just for this case.

Let us calculate g as a real positive root of the eq. (4):

$$7.75 a_4 g^4 - 10.524 a_3 g^3 + a_2 g^2 - 1 = 0 \qquad (4)$$

Then we calculate the feedback loop gain from eq. (5):

$$\begin{aligned} &B_4 = 1 - a_4 g^4, \\ &B_3 = 1 + 4.5(1 - B_4) - a_3 g^3, \\ &B_1 = 1 + 1.875(1 - B_4) + 1.75(1 - B_3) + 1.5 - a_1 g \end{aligned} \qquad (5)$$

This procedure provides B_2=0 (i.e. the bottom lead of the capacitor C_2 is connected to ground).

Then we have to account the finite rated bandwidth *GB* of the Op-Amp. Let us calculate small corrections:

$$d_1 = f_N K/a_1(GB),\ \ d_2 = 6.1 d_1 a_1/a_2 g, \qquad (6)$$
$$d_3 = 9.5 d_1 a_1/a_3 g^2,\ \ d_4 = 5.8 d_1 a_1/a_4 g^3 ,$$

where K is the gain of amplifier. In addition, a small extra term (i.e. a distant pole) appears in polynomial (1):

$$a_{(n+1)} s^{(n+1)} = (d_1 a_1 a_n) s^{(n+1)},\ \ n = 4 \qquad (7)$$

To eliminate the effect of *GB*, the coefficients a_k in eq. (1) must be adjusted with the corrections d_k from eq. (6):

$$a_k^* = a_k(1 - d_k),\ \ k = 1\ldots n \qquad (8)$$

Using a_k^* in eq. (2) – (5), we figure out corrected values g_*, B_k^*, choose elements R & C (see eq. (3)), etc.

Strictly speaking, we should introduce pre-distortion into transfer function to eliminate the error caused with the extra term (eq. (7)). In practice it is not needed in most cases. On the contrary, to employ corrections from eq. (6) is the routine procedure (except, may be, the case of non-precision filter for low-frequency range).

B. Example of the filter design

Let us design the Single Op-Amp Tchebyshev LPF concerned in Section II. Cutoff frequency is f_N =1 kHz, stopband f_a =2 kHz (attenuation a=36 dB), expected interference voltage in stopband is up to e_{INT} =150 V max. Given values: a_k in eq. (1), C =0.75 nF, GB =1.6 MHz, m =0.5, q =2.

Calculations are summarized in Table 1. Comments: resistors $r_2 = r_4$ =1 kOhm are chosen under the condition to provide $r_1//r_2 < 0.01R_3$ and $r_3//r_4 < 0.01R_4$ (the accurate account of adverse effect of these resistors is omitted)

The corrections: d_1=0.03%, the coefficient a_5=0.0038.

TABLE I
Calculation of parameters of 4-th-order LPF

Index k	4	3	2	1
Given a_k (eq. (1))	4.593	3.508	5.933	2.567
g=0.377; B_k from (5)	0.907	1.230		1.304
K=1.304; d_k from (6)	1.9%	1.6%	0.2%	
Corrected a_k^* (eq. (8))	4.505	3.453	5.920	2.566
g_*=0.378; B_k^* from (5)	0.908	1.228		1.304
$b_k = B_k^*/K$	0.696	0.942		
b_k element r_k, kOhm	1.00	2.29	1.00	16.1
q=2; from (2): C_k, nF	0.75	1.50	3.00	6.00
Eq. (3), (2): R_k^*, kOhm	561.4	280.7	140.3	70.2

For normal operation of Op-Amp the reasonable minor current i_{BACK} may be allowed only. It is seen, that this current in the schematic in Fig.4 is no more 0.23 mA for given e_{INT} =150 V. On the contrary, the current i_1 may reach to 2.1 mA. To protect the Op-Amp output we insert an emitter follower VT with current I_E >3 mA.

For practical realization we have to account the distribution of voltage produced by e_{INT} into RC-circuit, and to choose its elements correspondingly. For example, resistor R_1 is capable to endure the whole voltage e_{INT}, capacitor C_1 – around 30 V, etc.

IV. DESIGN OF 3-D-ORDER TRANSFER-ZERO-LPF FOR OPERATING AT HIGH INTERFERENCE

A. Unsuitable realization of the transfer zero

The most of commercially available integrated ARC-filters realize transfer zero via structure schematic in Fig.5.

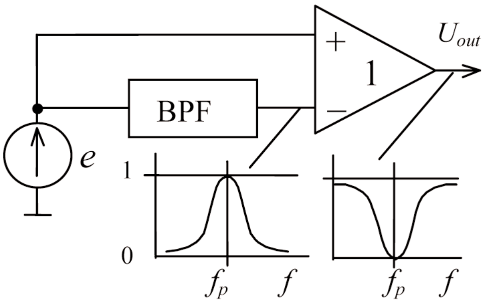

Fig.5. Structure schematic for the realization of transfer zero whith the band-pass filter

It is evident that the interference voltage at the frequency f_p is bound to be no more than the rated input signal level. This is the considerable defect of the building of transfer zero in such a way. The interference level in practice often is much more than a rated signal, and schematic in Fig. 3 is useless.

On the contrary, the schematic of the type in Fig. 1 can operate with interference/rated signal ratio up to 30 – 40 dB.

B. Procedure of the filter design

Let us consider the schematic in Fig.6.

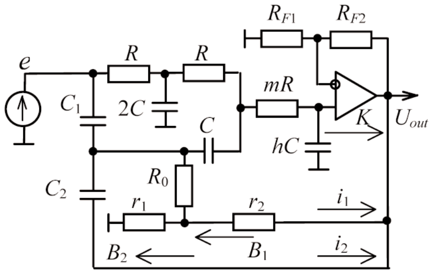

Fig.6. 3-d-order LPF with transfer zero

Its transfer function is:

$$T=U_{out}/e=[s^2(gf_N/f_Z)^2+1]/(a_3s^3+a_2s^2+a_1s+1), \quad (9)$$

where: s, g, f_N see in eq.(1),(3),

f_Z is given frequency of transfer zero,

a_k (a_k =1…3) are given coefficients.

We choose arbitrary values $g<1$, $h<0.5$ and C for technological reason. Then calculate parameters:

$$A=(gf_N/f_Z)^2,\ C_1=AC,\ C_2=C-C_1,\ m=a_3g^3/h; \quad (10)$$

$$\begin{aligned}&B_2=4a_3g^3+2h+1-a_2g^2,\\&B_1=0.5(a_3g^3+2h+4-a_1g),\\&K=B_2/(1-A),\ b_1=B_1/K\end{aligned} \quad (11)$$

(note that there is the condition $K \geq b_1$, for which chosen above values g and/or h may be changed),

$$\begin{aligned}&d_1=f_NK/a_1(GB),\ d_2=a_1(a_3g^3+2h+4)/a_2g,\\&d_3=d_1a_1(4a_3g^3+2h+1)/a_3g^2,\ a_4=d_1a_1a_3,\end{aligned} \quad (12)$$

and corrected values a_k^* (k =1…3) from eq. (8), B_k^*, K^*, b_1^* from eq. (11), etc.

C. Example of the filter design

Let us design the 3-d-order LPF with: f_N =1 kHz, f_Z =2 kHz; expected interference up to 150 V max on the zero frequency f_Z. Given transfer function is Tchebyshev fraction [8]: ripple of 0.5 dB, stopband frequency is f_a=1.77 kHz (attenuation is 24.7 dB) Given values: a_k, C=1.5 nF, GB=1.6 MHz. Chosen values: g=0.7, h=0.25.

Calculated parameters: from eq. (10) A=0.1225, C_1=184 pF, C_2=1316 pF. The other calculations are summarised in Table II.

TABLE II

Calculations of parameters of transfer-zero-LPF

Index k	3	2	1
Given a_k (eq. (10))	1 216	1 490	1.841
From eq (11): B_k and K=2.779		2.438	1.814
d_k from eq. (12)	0.8%	0.7%	0.08%
Corrected a_k^* (eq. (8))	1.206	1.479	1.826
m^*=1.656 from eq. (10)			
B_k^* from eq.(11)		2.430	1.818
K^*=2.769, b_1=0.656, r_k, kOhm		1.05	2.0
Eq. (3): R=151.6 kOhm; $R_0=R/2 - r_1//r_2$=75 kOhm			

Operating at e_{INTt}=150 v: current $i_1<0.16$ mA, $i_2<0.28$ mA. If $i_{BACK}=i_1+i_2<0.5$ mA is nevertheless “dangerous” for the Op-Amp, then emitter follower may be inserted (similar to Fig.4). Capacitor C_1 and the left resistor R in Fig.6 are bound be rated at 150 V.

V. DESIGN OF 4-TH-ORDER POLYNOMIAL HPF FOR OPERATING AT HIGH INTERFERENCE

A. Procedure of the filter design

The schematic of the HPF is presented in Fig.7.

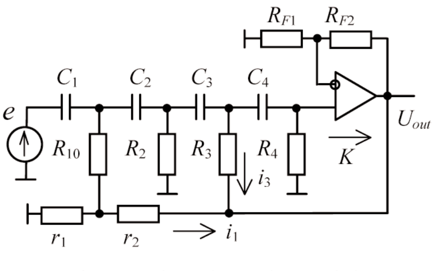

Fig.7. 4-th-order polynomial HPF

Its transfer function takes the form:

$$T=U_{out}/e=Ka_{4H}s^4/(a_{4H}s^4+\ldots a_{kH}s^k+\ldots a_{1H}s+1), \quad (13)$$

where: $s=i\omega/\omega_{NH}$, i is the square root of −1,

$\omega_{NH}=2\pi f_{NH}$, f_{NH} is cutoff frequency of HPF,

a_{kH} (k=1…4) are HPF coefficients:

$$a_{kH}=a_{(4-k)}/a_4,\ k=1\ldots4,\ a_0=1, \quad (14)$$

where a_k are the coefficients of the LPF prototype.

In schematic in Fig.7 the regular equal-valued-capacitor RC-circuit is employed. Its parameters:

$$R_k=m^{(4-k)}R,\ \ C_k=C,\ \ RC=1/\omega_{NH}. \qquad (15)$$

Let us solve nonlinear equation in x, take real positive root, and figure out values m and g:

$$4x^{10}+5x^8-2x^7a_{1H}/a_{4H}{}^{(1/4)}+4x^6-2x^5a_{1H}/a_{4H}{}^{(1/4)}+ \\ +x^4(1+a_{2H}/a_{4H}{}^{(1/2)})-1=0, \qquad (16) \\ m=x^2,\ \ g=m^{(3/2)}/a_{4H}{}^{(1/4).}$$

Calculate feedback loop gain:

$$B_3=K=1+2m(m^2+m+1)-a_{1H}\,g, \qquad (17) \\ B_1=1+3m^2(1-B_3)+2m(2m^2+1)-a_{3H}\,g^3/m^3,\ \ b_1=B_1/B_3,$$

and corrections:

$$d_1=f_{NH}K/a_{1H}(GB),\ \ d_2=d_1a_{1H}(a_{1H}+B_3/g)/a_{2H}, \\ d_3=d_1a_{1H}[a_{2H}+2m^2(m+1)B_3/g^2]/a_{3H}, \qquad (18) \\ d_4=d_1a_{1H}[a_{3H}+m^3(3m^2B_3+B_1)/g^3]/a_{4H}.$$

Then the procedure of calculation follows, the similar to that outlined above.

B. Example of the filter design

Let us design the 4-th-order Butterworth HPF for cutoff frequency f_N=300 Hz. Rated signal voltage is e=0.775 V. The filter is bound to endure an emergency voltage e_{INT}=110 V (155 V max) on the frequency f_{INT}= 60 Hz. Given values: coefficients a_k of the LPF-prototype, C=3 nF, GB=1.6 MHz.

Calculations are summarized in Table III. Corrections d_k are ignored inasmuch for low-sensitive transfer function of low frequency range filter.

TABLE III
Calculation of parameters of 4-th-order HPF

Index k	4	3	2	1
Given LP prototype a_k.	1.000	2.613	3.314	2.613
HPF a_k from eq.(14)	1.000	2.613	3.314	2.613
From (16): m=0.584, g=0.447				
From eq.(17): B_k and b_1=0.337		2.084		0.689
From (15): $R_{k,}$, kOhm	396	231	135	78.8
$b_1=r_1/(r_1+r_2)$, r_k, kOhm			1.97	1.00
$R_{10}=R_1-r_1//r_2$=78.1 kOhm				

Operating at high interference e_{INT}: “back” current $i_1\leq$ 0.06 mA (i_3 is negligible). Elements: C_1 is bound be rated for e_{INT}, C_2 and R_1 – for 14 V.

VI. CONCLUSION

Considered filters contained Op-Amps in number less as compared with another ARC-filters, what means low consumption, simple network and technology, hence increased reliability and low cost

The main advantage of these filters contains in operating at very high input interference voltage in stopband, what is important in instrumentation. Nevertheless filters of this type are not used widely, probably because of accurate design is necessary to obtain successfully operating device. The methodology of synthesis and design is briefly presented. This paper may be considered as an introduction to the problem. Nevertheless, the author expects that this work may be useful for specialists.

The methodology of design may be also found in [9].

ACKNOWLEDGMENT

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APPENDICES

Appendix 1. ARC-ANALOG DEVICES ARE BASIC BLOCKS FOR HIGH TEMPERATURE APPLICATIONS

Digital telemetering systems involves numerous chips containing large number of transistors. It is unreal for High Temperature (HiT) Devices. On the contrary, the telemetry with frequency division multiplex, based upon ARC-analog devices, requires not many active elements. ARC-circuits similar to the considered in this paper, are especially convenient for this purpose. As an example, draft project of the telemetry for deep submerged pumps in oil wells was considered. It includes 34-40 FETs with R & C inside the HiT block.

Appendix 2. The matched Single Op-Amp ARC-thin-film filter-conditioner for installation into accelerometer is draft designed. The total capacitance is 800 pF, total resistance is 320 kOhm.