DETECTING OF THE DETERMINISTIC SIGNALS ON A PHON OF GAUSSIAN NOISE WITH USE OF NEURAL NETWORK LOGIC BASIS

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Abstract – **In this paper the neural network solving a problem of detection of the deterministic signal on a phon of noise in conditions of parametrical aprioristic uncertainty is described. The neural network is constructed proceeding from the assumption that the noise signal submits to the normal law of distribution. Besides the algorithm of training of the constructed network that is modification of error back propagation algorithm is offered.**

Index terms – **Attitude of credibility, density of probability, normal law of distribution, deciding rule, threshold of decision-making, neuron, neural network, training sample, gradient method of optimization, algorithm of neural network training.**

I. Introduction

One of the problems at the organization of communication and radar systems is the problem of detection of a useful signal on a noise phon. Thus either a noise signal or a signal representing a mix of useful and noise signal can act on an input of the receiver. Detection consists in decision-making on, whether the given realization of an entrance signal contains a useful signal (event H_1), or does not contain (event H_0). Thus the decision of a problem of detection demands presence of algorithm of decision-making on the one hand, and also creations of the device realizing this algorithm - with another.

II. Algorithm of decision-making

The problem of detection is solved means of the theory of statistical decisions. Within the framework of this theory for detection the deciding rule based on formation of the attitude of credibility L and his comparison with a certain threshold c, by a named threshold of decision-making, is used [1, 2]:

$$L = \frac{w_n(x_1, x_2, \ldots, x_n | H_1)}{w_n(x_1, x_2, \ldots, x_n | H_0)} \geq c, \qquad (1)$$

where $w_n(x_1, x_2, \ldots, x_n | H_0)$ – conditional joint n-dimensional density of probability of selective values $x_1, x_2, \ldots, x_n$ provided that there was event H_0, and $w_n(x_1, x_2, \ldots, x_n | H_1)$ – conditional joint n-dimensional density of probability of selective values $x_1, x_2, \ldots, x_n$ provided that there was event H_1.

Value of a threshold c is defined by the chosen criterion of quality.

Thus at performance of a condition (1) it is made a decision that there was event H_1 and H_0 otherwise.

Values $x_1, x_2, \ldots, x_n$ are the parameters of an entrance signal.

Provided that parameters $x_1, x_2, \ldots, x_n$ are a set of jointly independent casual values, their conditional joint density of probability can be submitted by products:

$$w_n(x_1, x_2, \ldots, x_n | H_j) = \prod_{i=1}^{n} w_1(x_i | H_j), \; j = 0, 1, \qquad (2)$$

where $w_1(x_i | H_j)$ is a one-dimensional conditional density of probability of value x_i under condition of occurrence of event H_j.

Let's consider a situation when the condition of parametrical aprioristic uncertainty is satisfied. In this case the kind of functions of density of probability $w_1(x_i | H_j)$ is known but parameters of this distribution are not known.

Let parameters $x_1, x_2, \ldots, x_n$ submit to the normal law of distribution. In this case one-dimensional conditional density of probability of parameter x_i are represented by expressions:

$$w_1(x_i | H_j) = \frac{1}{\sqrt{2 \cdot \pi} \cdot \sigma_{ij}} \cdot \exp\left\{ -\frac{(x_i - a_{ij})^2}{2 \cdot \sigma_{ij}^2} \right\}, \quad j = 0, 1, \qquad (3)$$

where a_{i0} – average value of parameter x_i provided that has arisen event H_0, a_{i1} – average value of parameter x_i provided that there was event H_1, σ_{i0}^2 – a dispersion of parameter x_i provided that there was event H_0, σ_{i1}^2 – a dispersion of parameter x_i provided that there was event H_1.

Substituting expression (3) and (2) in (1) and finding the logarithm from the received attitude of credibility L we shall receive deciding rule of a kind:

$$\sum_{i=1}^{n}\left(\left(\frac{1}{2\cdot\sigma_{i0}^{2}}-\frac{1}{2\cdot\sigma_{i1}^{2}}\right)\cdot x_{i}^{2}+\left(\frac{a_{i1}}{\sigma_{i1}^{2}}-\frac{a_{i0}}{\sigma_{i0}^{2}}\right)\cdot x_{i}\right)+$$
$$+\left(-\ln(c)+\ln\left(\mathrm{P}_{i=1}^{n}\frac{\sigma_{i0}}{\sigma_{i1}}\right)+\sum_{i=1}^{n}\left(\frac{a_{i0}}{2\cdot\sigma_{i0}^{2}}-\frac{a_{i1}}{2\cdot\sigma_{i1}^{2}}\right)\right)\geq 0. \qquad (4)$$

In conditions of parametrical aprioristic uncertainty the parameters of distribution of casual variables x_i – a_{i0}, $a_{i1}, \sigma_{i0}^2, \sigma_{i1}^2$ – are unknown, and they are subject to definition. The threshold of decision-making c is unknown also.

Expression (4) can be presented as:

$$\sum_{i=1}^{n}\left(a_i\cdot x_i^2+b_i\cdot x_i\right)+\lambda\geq 0, \qquad (5)$$

where a_i, b_i and λ represent the values dependent on parameters of functions of density of probability (2), i.e. also are unknown.

III. Realization of algorithm of decision-making by means of neural network logic basis

The algorithm of decision-making submitted by a rule (5), can be realized by means of a two-layer neural network in which are used the neurons with linear and square-law functions of inclusion and also function of inclusion in the form of threshold function [4]. The structure of such network is submitted on Fig.1.

On Fig.1: F_1 – neuron realizing expression $F_1(x)=x$. F_2 – neuron with square-law function of activation, i.e. neuron realizing expression $F_2(x)=x^2$.

Neuron F_0 realizes threshold function of a kind:

$$F_0(x)=\begin{cases}\infty, & x\geq 0\\ 0, & x<0\end{cases}. \qquad (6)$$

All neurons of the first layer are neurons with one input, i.e. carry out function of touch neurons. These neurons implement functional transformation of entrance parameters $x_1, x_2, \ldots, x_n$ with use of functions F_1 or F_2. Weight factors of neurons of this layer are constant equal to unit and do not vary during training a neural network.

The signal equivalent to unit signal acts on one of neurons of the first layer.

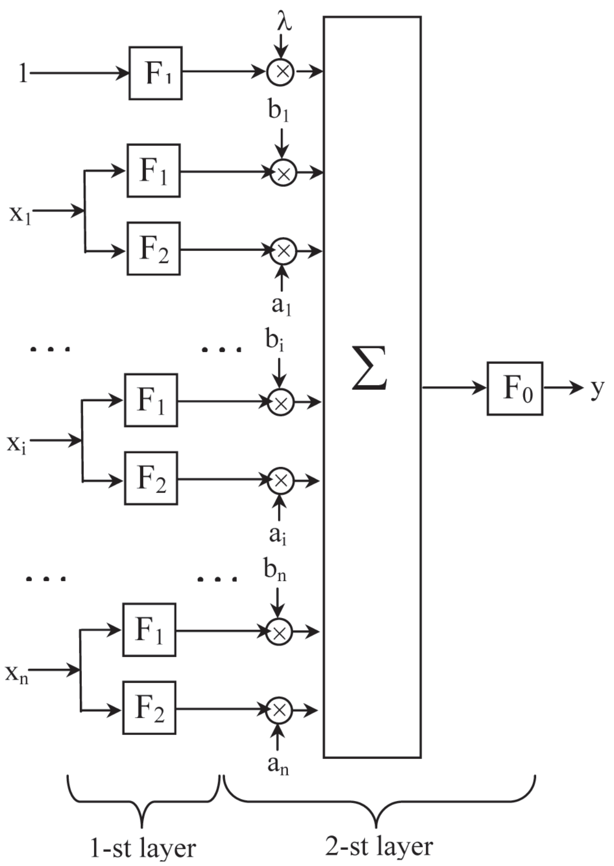

Fig. 1. The two-layer neural network realizing the deciding rule (5).

At the second layer of a network there is only one neuron F_0 on which output the signal $y=\infty$ is formed in a case if there is event H_1. The signal $y=0$ on an output of neuron F_0 means that there was event H_0.

Value $y=\infty$ cannot be received for one real neuron. Such possible value y is used to emphasize only that fact that concrete values y is not important. Important only that y is distinct from zero, i.e. was the situation corresponding to event H_1 which excludes event H_0. Therefore at practical realization of a network on fig.1 under value $y=\infty$ it is possible to mean any value $y\neq 0$.

Signals from all neurons of the first layer act on an input of neuron F_0. Neuron F_0 implements operation of the weighed summation of signals of neurons of the first layer with weight factors a_i, b_i and λ ($i = 1 \ldots n$). Thus the rule (5) is realized.

III. Algorithm of neural network training

As weight factors ai, bi and λ are unknown the network requires preliminary adjustment or training.

For adjustment of a network set of training samples (X_α, Y_α), α = 1 … p, where X_α is a vector of entrance parameters of sample with number α, Y_α is a value of output parameter y corresponding to the given sample X_α, and p is the general number of training samples, is used.

The constructed neural network is a two-layer network without feedback for which training use of described below modification of error back propagation algorithm is possible [3].

Step 1. Initial values of weight factors a_i, b_i и λ (i= 1 … n) undertake equal to casual values. On the other hand, if there is an opportunity even approximately to estimate values of these weight factors it will allow to reduce time of training of a network as training will begin near to the points close to optimum.

Step 2. Entrance image X_α is showed to a network, in result the equal or not equal Y_α output image y is formed. Thus neurons consistently from a layer to a layer function under the following expressions.

First layer:

$$x_{1i} = F_1(x_i); \quad x_{2i} = F_2(x_i); \quad x_\lambda = F_1(1). \tag{7}$$

Second layer:

$$y_1 = \sum_{i=1}^{n}(b_i \cdot x_{1i} + a_i \cdot x_{2i}) + \lambda \cdot x_\lambda; \quad y = F_0(y_1). \tag{8}$$

Step 3. The functional of a square-law error for this entrance image is:

$$E = \frac{1}{2}\cdot(y - Y_\alpha)^2. \tag{9}$$

On this step it is required to achieve the functional has become equal to zero for this entrance image X_α. E is equaled to zero only when $y = Y_\alpha$.

The classical gradient method of optimization will consist in iterative specification of argument according to the expression:

$$V_j(t+1) = V_j(t) - h \cdot \frac{\partial E}{\partial V_j}, \tag{10}$$

where Vj in our case are all weight factors of a neural network.

Thus, during training a network it is necessary to make at each iteration change of weight factors according to the expression (10) where t is number of the current iteration, and (t+1) is number of the following iteration.

Having taken advantage of the expression of implicit differentiation we shall receive:

$$\frac{\partial E}{\partial b_i} = \frac{\partial E}{\partial y}\cdot\frac{\partial y}{\partial y_1}\cdot\frac{\partial y_1}{\partial b_i} = (y - Y_\alpha)\cdot x_{1i} = (y - Y_\alpha)\cdot x_i, \tag{11}$$

$$\frac{\partial E}{\partial a_i} = \frac{\partial E}{\partial y}\cdot\frac{\partial y}{\partial y_1}\cdot\frac{\partial y_1}{\partial a_i} = (y - Y_\alpha)\cdot x_{2i} = (y - Y_\alpha)\cdot x_i^2, \tag{12}$$

$$\frac{\partial E}{\partial \lambda} = \frac{\partial E}{\partial y}\cdot\frac{\partial y}{\partial y_1}\cdot\frac{\partial y_1}{\partial \lambda} = (y - Y_\alpha)\cdot x_\lambda = (y - Y_\alpha)\cdot 1. \tag{13}$$

At a deducing of expressions (11) – (13) it has been taken into account, that:

first, $\frac{\partial F_0(x)}{\partial x} = F_0(x)$,

and second, $(y - Y_\alpha)\cdot y = (y - Y_\alpha)$.

Final expressions for correction of weight factors will be as follows:

$$b_i(t+1) = b_i(t) - H \cdot x_i, \tag{14}$$

$$a_i(t+1) = a_i(t) - H \cdot x_i^2, \tag{15}$$

$$\lambda(t+1) = \lambda(t) - H \cdot 1. \tag{16}$$

Here $H = h \cdot (y - Y_\alpha)$.

The difference $(y - Y_\alpha)$ can accept one of three values: –∞, 0, +∞. For maintenance of convergence of a method it forces to select h so that product $H = h \cdot (y - Y_\alpha)$ was final. Let's:

$$h = \frac{\delta}{|y - Y_\alpha|}, \tag{17}$$

where δ is positive real number.

Then H will accept one of three values. When $y = Y_\alpha$: H = 0. When $y = \infty$ и $Y_\alpha = 0$: $H = \delta$. When y = 0 и $Y_\alpha = \infty$: $H = -\delta$. I.e. in a case if the output image of a neural network does not correspond to an entrance image, there will be a change of weight factors, i.e. there will be an adjustment of a network for the entrance image.

Thus, H is meaningful of training rate and gets out small enough (usually less unit) for convergence of a method.

The third step comes to an end when at the next iteration condition E = 0 is satisfied.

Step 4. Steps 1 - 3 repeat for all training samples. Training comes to the end when for all samples of the current pass condition E = 0 was satisfied, and corrections of weight factors were not made.

V. Conclusions

Thus, at the decision of problems of detection of the deterministic signal on a noise phon probably use of neural networks. Application of neural networks allows solving a problem of construction of realizing decision-making rule devices when there are conditions of parametrical aprioristic uncertainty, i.e. when there are no aprioristic data on parameters of distribution of parameters of a detected signal.

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