CHAOS ORIGINATION IN DISCRETE SYSTEMS BY THE EXAMPLE OF LOGISTIC EQUATION

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Abstract – **The report deals with the problem of chaos origination in discrete systems. Logistic equation have been analyzed. The criteria of. chaos origination have been implemented to define the parameter value area, where the system output signal is chaotic. The dependence of statistical characteristics on the signal realization length have been considered.**

Index terms – **Dynamic chaos, discrete system, Lyapunov index, bifurcation diagram.**

I. Introduction

Recently, the discrete-time generators of chaotic signal find more applications in communication and control systems. Continuous chaotic signal may appear only in nonlinear dynamic systems with the order of 3 or more. Still, some chaotic modes originate in nonlinear discrete systems with the order of 2 or even 1. The simplicity of mathematical description of such systems makes it possible to facilitate the analysis and design.

Determining the range of parameters values, which gives the system the chaotic dynamics, is one of the most significant problems. Generally used criteria are bifurcation diagram and Lyapunov index.

Another important matter is the analysis of influence of signal realization length on statistical characteristics of output signal.

II. Logistic equation

Let's consider nonlinear discrete system, described with well known logistic equation, which is widely used in implementation of digital communication systems.

The system model is given by:

$$x_n = \alpha x_{n-1}\left(1 - x_{n-1}\right), \qquad (1)$$

where α – parameter. The value area of α is $\left[0,4\right]$. Initial condition x_0 of the system is also limited: its value area is $\left[0,1\right]$.

To define the parameter value area, where the system output signal is chaotic, we should use some chaos criteria. Bifurcation diagram and Lyapunov index are in general use.

Bifurcation diagram is the dependence of steady-state oscillation magnitude on parameters of nonlinear system [1]. To plot it, we should calculate the value of signal x_N, where N ? 1 (ideally, $N \to \infty$), for each values of α and x_0 in its value areas. Bifurcation diagram is shown in Figure 1 ($N = 300$ was taken for calculations).

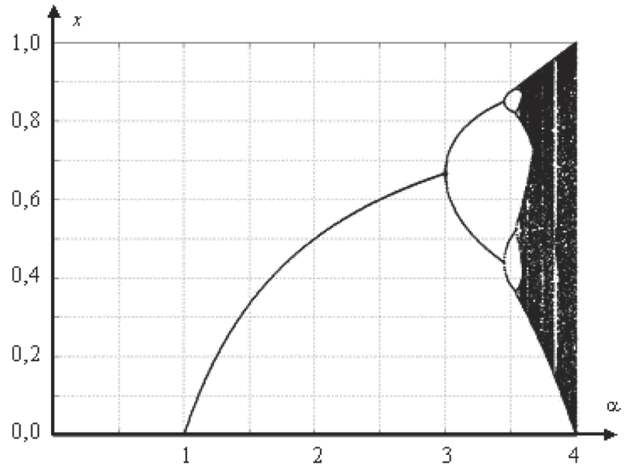

Figure 1. Bifurcation diagram
for logistic equation (1)

If $\alpha < 3$ (see Figure 1), the system has one attracting point, and all trajectories converge to this point, which is periodic with period $T = 1$ [2]. Under $\alpha = 3$, the first bifurcation of period doubling takes place. The system has now stable cycle with period $T = 2$. Under $\alpha > 3,56994$, the system may have chaotic oscillations.

One of the characteristic properties of chaotic modes is the instability of every trajectory belonging to chaotic attractor. Lyapunov index turned out to be a very good quantitative measure of this instability. In nonlinear systems Lyapunov indices characterize the dispersing rate of infinitely near trajectories.

Dynamic chaos corresponds to the instability of every separate trajectory, i.e. the presence of at least one positive Lyapunov index. In this case the corresponding attractor is called "chaotic". Lyapunov index "characterizes the dispersing rate of chaos realizations with close initial conditions [3].

If the discrete chaos equation is given by

$$x_{i+1} = f\left(x_n\right),$$

Lyapunov index is calculated as

$$\lambda(x_0) = \lim_{N\to\infty} \frac{1}{N} \ln\left|\frac{df^N(x_0)}{dx_0}\right|, \qquad (2)$$

where $f^N(x_0)$ – function of N-th chaos sample calculation, x_0 – initial condition.

The exact computation in accordance with (2) is not always possible since function $f^N(x_0)$ often can not be found. That's why in [3] a modification of this formula for experimental (numerical) determination of Lyapunov index was suggested:

$$\lambda(x_0) = \lim_{N\to\infty} \frac{1}{N} \sum_{i=0}^{N-1} \left|f'(x_i)\right|,$$

where $f'(x)$ – the derivative of $f(x)$ in respect to x.

Lyapunov index for logistic equation (1) was calculated subject to $N = 400$ (see Figure 2). No dependence of Lyapunov index λ on initial conditions x_0 was found. Values of parameter α, where λ is positive, correspond to chaotic modes of the system.

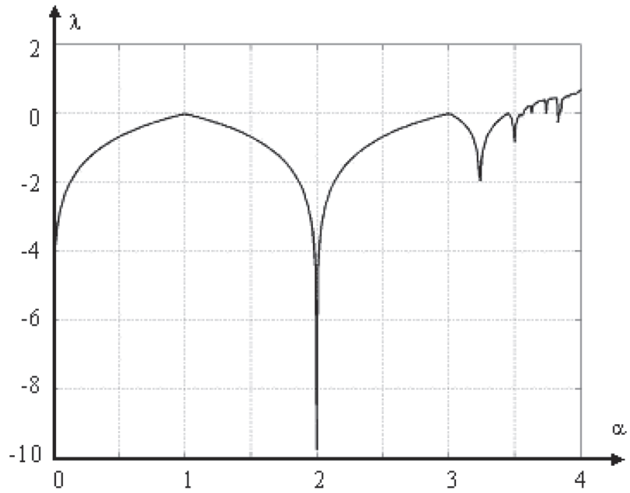

Figure 2. Lyapunov index
for logistic equation (1)

As we can see, the correlation between bifurcation diagram and Lyapunov index $\lambda(\alpha)$ is beyond any doubts.

The example of chaotic process ($\alpha = 3{,}8$; $x_0 = 0{,}4$, realization length 100) is shown in Figure 3.

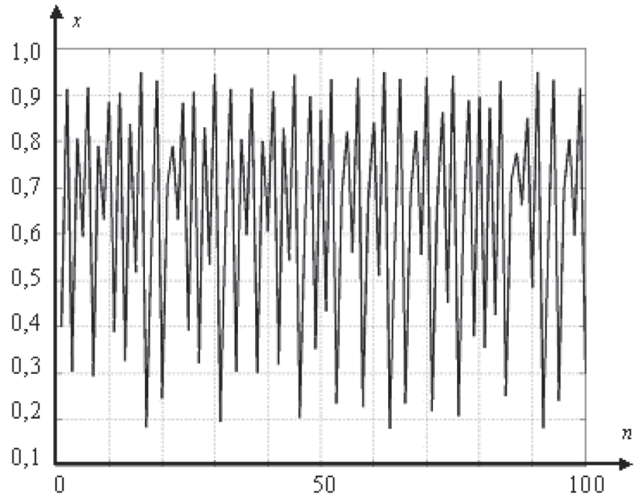

Figure 3. Chaotic mode
for logistic equation (1)
($\alpha = 3{,}8$; $x_0 = 0{,}4$,realization length 100)

III. Statistical characteristics

To analyze the possibility of using logistic equation model (1) in chaos generators we should also pay attention to its statistical characteristics.

Figures 4 and 5 demonstrate the dependence of mean value and variance of the signal (1) on the length of realization n.

It is clear that these parameters are slightly dependent on initial conditions, if the realization length is big enough. Hence there is a necessity of using "long" realizations of chaotic signal, since such characteristics are less dependent on modulating parameters. Nevertheless, it's worth noting that "long" realization of chaotic signal in digital communications systems bring about significant loss of data transfer rate.

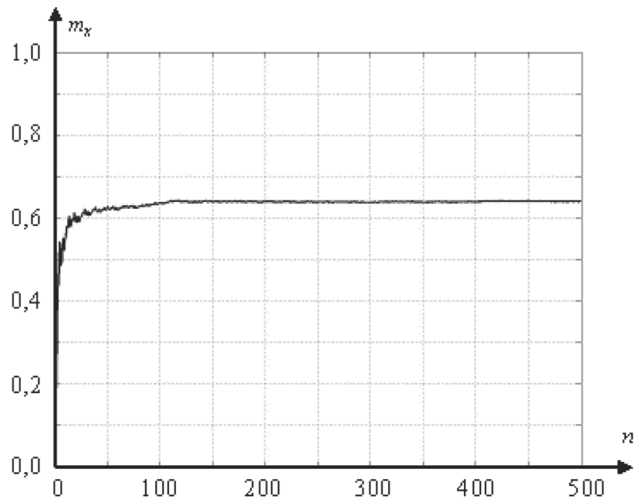

Figure 4. Mean value m_x
($\alpha = 3{,}8$; $x_0 = 0{,}1$)

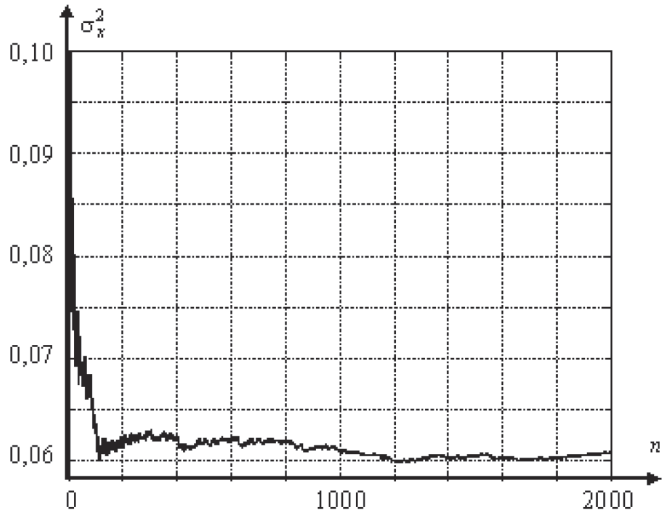

Figure 5. Variance σ_x^2
($\alpha = 3{,}8;\ x_0 = 0{,}1$)

IV. Conclusions

Classical criteria have been used to define the parameter range, where the discrete system, described by logistic equation, is in chaotic mode. Thus, having this information, we can make chaos generator system, based on the logistic model for using in communication systems. The simplicity of logistic equation also gives us possibility to get acquainted with the phenomena of chaos. A dependence of statistical characteristics on signal length realization of the system has been calculated.

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