REFLECTION OF ELECTROMAGNETIC WAVE FROM THE BORDER OF ELASTIC ENVIRONMENT FROM PIEZOACTIVE MATERIAL

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DOI: 10.36724/2664-066X-2020-6-5-2-5

ABSTRACT

A plane electromagnetic wave incident on the interface of a dielectric medium made of a piezoactive material leads to the excitation of elastic shear waves. Piezoelectric material is assumed to have 6mm class hexagonal symmetry. The case of a half-space and a finite layer is considered. Coefficients of reflection and refraction are determined. In the problem of an electromagnetic wave incidence on a half-space made of a piezoelectric material of hexagonal symmetry, class 6 mm, the nature of the piezoelectric effect influence on the transparency condition is established. In the case of reflection from a finite piezolayer, reflection coefficients are determined for different variants of boundary conditions.

KEYWORDS: *Electromagnetic wave, piezoelectric materia, electroelastic waves.*

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The influence of the electromagnetic field on the nature of the reflection and refraction of shear elastic waves in a quasi-static formulation was studied in [1-5]. Piezoelectric materials of cubic and hexagonal symmetry were considered. Particular problems of reflection of shear electroelastic waves without a quasi-static approximation for the equations of the electromagnetic field were solved in [6, 7].

Naturally, in problems of excitation of electroelastic waves by means of an electromagnetic wave incident on an elastic medium, it is necessary to use the exact equations of electrodynamics.

1. In a rectangular Cartesian coordinate system (x, y, z) half-space properties

$$-\infty < x < 0,\ -\infty < y < \infty, -\infty < z < \infty$$

are identified with the properties of vacuum. Half-space occupying an area

$$0 < x < \infty,\ -\infty < y < \infty,\ -\infty < z < \infty$$

is an elastic piezoelectric of hexagonal symmetry of class 6 mm. From half-space $-\infty < x < 0$ to the interface $x = 0$ falling transversely polarized flat $\left(\partial/\partial z = 0\right)$ electromagnetic wave with electric field components $E_1^{(1)}$, $E_2^{(2)}$ and magnetic field $H_3^{(1)}$. The electrodynamic equations for incident and reflected electromagnetic waves are conveniently used in the form

$$\Delta H_3^{(1)} = \varepsilon_0 \mu_0 \frac{\partial^2 H_3^{(1)}}{\partial t^2},$$
$$\frac{\partial E_1^{(1)}}{\partial t} = \frac{1}{\varepsilon_0}\frac{\partial H_3^{(1)}}{\partial y},\ \frac{\partial E_2^{(1)}}{\partial t} = -\frac{1}{\varepsilon_0}\frac{\partial H_3^{(1)}}{\partial x} \tag{1}$$

where

$$\Delta \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2},\ \varepsilon_0 \mu_0 = \frac{1}{c_0^2} \tag{2}$$

c_0 – speed of light propagation in a vacuum

Electroelasticity equations for piezoelectric medium $0 < x < \infty$, without quasi-static approximation, the following [7, 8]

$$\Delta w = \frac{1}{c_t^2}\frac{\partial^2 w}{\partial t^2},\ \Delta H_3 = \frac{1}{c^2}\frac{\partial^2 H_3}{\partial t^2}$$
$$\frac{\partial E_1}{\partial t} = \frac{1}{\varepsilon_1}\frac{\partial H_3}{\partial y} - \frac{e_{15}}{\varepsilon_1}\frac{\partial^2 w}{\partial x \partial t},\ \frac{\partial E_2}{\partial t} = -\frac{1}{\varepsilon_1}\frac{\partial H_3}{\partial x} - \frac{e_{15}}{\varepsilon_1}\frac{\partial^2 w}{\partial y \partial t} \tag{3}$$

Here

$$c_t^2 = \frac{c_{44}}{\rho}(1+\chi),\ c^2 = \frac{1}{\varepsilon_1 \mu},\ \chi = \frac{e_{15}^2}{\varepsilon_1 c_{44}} \tag{4}$$

w – elastic displacement perpendicular to the plane of wave propagation $(x, y), c_{44}$ – shear modulus, ρ – medium material density, e_{15} – piezomodule, ε_1 and μ – dielectric and magnetic permeability of the medium, χ – electromechanical coupling coefficient.

As the main variant of the boundary conditions on the interface plane $x = 0$ the equality of tangential stress to zero and continuity of tangential components of electric vectors and magnetic fields are assumed

$$\sigma_{13} = 0,\ E_2^{(1)} = E_2,\ H_3^{(1)} = H_3 \quad \text{при } x = 0 \tag{5}$$

In the future, other variants of the boundary conditions will also be considered.

From the solutions of equations (1) for an electromagnetic wave incident on the interface, the following expressions are obtained

$$H_{3n}^{(1)} = A \exp i\left(\omega t - kx - k_2 y\right),\ k = \sqrt{\frac{\omega^2}{c_0^2} - k_2^2}$$
$$E_{1n}^{(1)} = -\frac{k_2}{\varepsilon_0 \omega} A \exp i\left(\omega t - kx - k_2 y\right), \tag{6}$$
$$E_{2n} = \frac{k}{\varepsilon_o \omega} A \exp i\left(\omega t - kx - k_2 y\right).$$

In accordance with (6), the electromagnetic wave reflected from the boundary will have the form

$$H_{30}^{(1)} = B \exp i\left(\omega t + kx - k_2 y\right)$$
$$E_{10}^{(1)} = -\frac{k_2}{\varepsilon_0 \omega} B \exp i\left(\omega t + kx - k_2 y\right), \tag{7}$$
$$E_{20}^{(1)} = -\frac{k_2}{\varepsilon_0 \omega} B \exp i\left(\omega t + kx - k_2 y\right)$$

Where is the amplitude B is the required quantity.

2. he fall of an electromagnetic wave on the boundary of a medium made of a piezoelectric material leads to the excitation of coupled elastic and electromagnetic waves in this medium. According to the first two equations of system (3), the magnetic field of the refracted electromagnetic wave and the excited elastic shear wave are determined as follows

$$H_3 = F \exp i\left(\omega t - p_2 x - k_2 y\right)$$
$$w = C \exp i\left(\omega t - p_1 x - k_2 y\right) \tag{8}$$

where

$$p_2 = \sqrt{\frac{\omega^2}{c^2} - k_2^2},\ p_1 = \sqrt{\frac{\omega^2}{c_t^2} - k_2^2}, c^2 = \frac{1}{\varepsilon \mu} \tag{9}$$

Arbitrary constants F and C must be determined from the boundary conditions.

Substitution of (8) into the third and fourth equations of system (3) determines the components of the electric field

$$E_1=-\frac{K_2}{\varepsilon_1\omega}F\exp i\left(\omega t-p_2x-k_2y\right)+$$
$$+i\frac{p_1e_{15}}{\varepsilon_1}C\exp i\left(\omega t-p_1x-k_2y\right) \tag{10}$$
$$E_2=-\frac{p_2}{\varepsilon_1\omega}F\exp i\left(\omega t-p_2x-k_2y\right)+$$
$$+i\frac{k_2e_{15}}{\varepsilon_1}C\exp i\left(\omega t-p_1x-k_2y\right).$$

Satisfaction of boundary conditions (12) with the condition

$$E_2^{(1)}=E_{2h}^{(1)}+E_{2v}^{(1)},\; H_3^{(1)}=H_{3n}^{(1)}+H_{30}^{(1)} \tag{11}$$

Leads to a system of algebraic equations for the sought amplitudes B, F, C

$$A-B=\frac{\varepsilon_0\omega}{k}\left(\frac{p_2}{\varepsilon_1\omega}F+i\frac{e_{15}k_2}{\varepsilon_1}C\right)$$
$$A+B=F \tag{12}$$
$$ip_1c_{44}\left(1+\chi\right)C-\frac{k_2e_{15}}{\varepsilon_1\omega}F=0$$

In particular, from system (12), the amplitude of the reflected magnetic field is determined as follows

$$B=\frac{\left(1-\chi\right)p_1\left(\varepsilon_1k-\varepsilon_0p_2\right)-\varepsilon_0\chi k_2^2}{\left(1+\chi\right)\left(p_1\right)\left(\varepsilon_1k+\varepsilon_0p_2\right)+\varepsilon_0\chi k_2^2}A \tag{13}$$

The equality to zero of numerator of expression (13) determines the conditions for the transparency of media relative to each other. By choosing the coefficient of electromechanical coupling χ you can control the degree of transparency

The condition of equality to zero of the denominator of expression (13) can be transformed to the form

$$\left(\varepsilon_1\sqrt{\theta_0\eta-1}+\varepsilon\sqrt{\theta_\eta-1}\right)\sqrt{\eta-1}+\frac{\varepsilon\chi}{1+\chi}=0 \tag{14}$$

where

$$\theta_0=\frac{c_t^2}{c_i^2}<<1,\;\;\theta=\frac{c_t^2}{c^2}<<1,\;\eta=\frac{\omega^2}{k_2^2c_t^2} \tag{15}$$

Assuming $\eta<1$ (the angle of incident wave inclination with interface is small) equation (14) has a solution of the Gulyaev-Bluestein surface wave type. From (14) $\theta_0\eta<<1,\;\theta_\eta<<1$ we obtain a solution for the dimensionless parameter of the Gulyaev-Bluestein phase velocity [4]

$$\eta=1-\frac{\varepsilon^2\chi^2}{\left(\varepsilon_1+\varepsilon\right)^2\left(1+\chi\right)^2} \tag{16}$$

From (14) and (13) it follows that at normal incidence of an electromagnetic wave $\left(k_2=0\right)$ the piezoelectric effect does not affect the reflected wave and does not excite an elastic wave.

Consideration of other variants of boundary conditions at $x=0$ leads to the conclusion that there is no influence of piezoelectric effect. For boundary conditions

$$\sigma_{xz}=0,\;E_2^{(1)}=0\text{ at }x=0 \tag{17}$$

It turns out $B=A, F=C=0$ if

$$\sigma_{xz}=0,\;H_3^{(1)}=0\text{ at }x=0 \tag{18}$$

It turns out $B=-A,\;\; F=C=0$

Obviously, there will be no piezoelectric effect if one of the boundary conditions assumes zero elastic displacement $\left(w=0\right)$.

3. Let an electromagnetic wave fall on a layer of piezoelectric material with a thickness h. Solution of the first two equations from system (3) for region $0<x<h$ are presented as follows

$$w=f\left(x\right)\exp i\left(\omega t-k_2y\right),$$
$$H_3=g\left(x\right)\exp i\left(\omega t-k_2y\right). \tag{19}$$

The requirement that (19) satisfies Eqs. (3) leads to the solutions

$$f\left(x\right)=C_1\sin p_1x+C_2\cos p_1x, g\left(x\right)=$$
$$=D_1\sin p_2x+D_2\cos p_2x \tag{20}$$

where $C_1, C_2,\; D_1, D_2$ – arbitrary constants.

The components of the electric field are determined from third and fourth equations of system (3), taking into account (19) and (20)

$$E_1=-\frac{k_2}{\varepsilon_1\omega}\left(g+\frac{e_{15}\omega}{k_2}f'\right)\exp i\left(\omega t-k_2y\right)$$
$$E_2=-\frac{i}{\varepsilon_1\omega}\left(g'+e_{15}\omega k_2f\right)\exp i\left(\omega t-k_2y\right) \tag{21}$$

where the prime means differentiation with respect to x.

Substitution (8), (10), taking into account (11) and (19), (21), taking into account (20) into the boundary conditions (5) leads to a system of equations for the sought constants

$$\varepsilon_1k\left(A-B\right)=\varepsilon\left(p_2D_1+e_{15}\omega k_2c_2\right),\;A+B=D_2$$
$$C_{44}\left(1+\chi\right)\varepsilon_1\omega p_1C_1+k_2e_{15}D_2=0 \tag{22}$$

On the second plane that bounds the layer, various variants of the boundary conditions are possible. The most general is the boundary conditions, when at $x=h$ the tangential stress is zero and the electromagnetic field is continuous.

Limit cases are considered here

$$w=0,\;E_2=0\text{ при }x=h \tag{23}$$

and

$$w=0,\;H_3=0\text{ при }x=h \tag{24}$$

In the case of boundary conditions (23), according to (19) - (21), we obtain

$$\begin{aligned} & C_1 \sin p_1 h + C_2 \cos p_1 h = 0 \\ & p_2 D_1 \cos p_2 h - p_2 D_2 \sin p_2 h = 0 \end{aligned} \quad (25)$$

The system of equations (22), (25) determines the constants B, C_1, C_2, D_1, D_2 through the amplitude of the incident wave A. From this system, in particular for amplitude of reflected wave, we obtain

$$\begin{aligned} B = & \left(\varepsilon_1 p_1 k - \varepsilon p_1 p_2 tgp_2 h - \chi \varepsilon k_2^2 tgp_1 h\right) \times \\ & \times \left(\varepsilon_1 p_1 k + \varepsilon p_1 p_2 tgp_2 h + \chi \varepsilon k_2^2 tgp_1 h\right)^{-1} A \end{aligned} \quad (26)$$

From (26) it follows that the numerator and denominator of expression for amplitude of the reflected electromagnetic wave vanishes at $p_1 = 0$ (and $k_2^2 = \omega^2 c_t^{-2}$). By dividing into p_1 and limiting the specified uncertainty is eliminated

$$\begin{aligned} B = & \left(\varepsilon_1 k - \varepsilon p_2 tgp_2 h - \chi \varepsilon k_2^2 h\right) \times \\ & \times \left(\varepsilon_1 k + \varepsilon p_2 tgp_2 h + \chi \varepsilon k_2^2 h\right)^{-1} A \end{aligned} \quad (27)$$

Equality of the expression numerator to zero gives the condition under which electromagnetic wave is not reflected. In the approximation of a thin layer $\left(p_2 h\right)^2 << 1$ this condition takes the form

$$\varepsilon_1 \sqrt{\frac{\omega^2}{c_0^2} - K_2^2} - \varepsilon h \left(\frac{\omega^2}{c^2} - k_2^2 + \chi K_2^2 \right) = 0 \quad (28)$$

It follows from (28) that the electromechanical coupling coefficient χ can make the layer transparent to the electromagnetic wave.

If, provided $c_0^2 < c^2$ angle of incidence of the wave on the interface $x = 0$ choose $k_2^2 = \omega^2 c^{-2}$, the transparency condition will be

$$\frac{\omega}{c} = \frac{\varepsilon_1}{\chi \varepsilon h} \sqrt{\frac{c^2}{c_0^2} - 1} \quad (29)$$

In the case of boundary conditions (24), system of equations similar to equations (25) will be

$$\begin{aligned} & C_1 \sin p_1 h + C_2 \cos p_1 h = 0 \\ & D_1 \sin p_2 h + D_2 \cos p_2 h = 0 \end{aligned} \quad (30)$$

Equations (30) together with equations (22) constitute a system of five equations for determining the constants B, C_1, C_2, D_1, D_2 by the amplitude of the incident wave A, B. In particular, for the amplitude of reflected electromagnetic wave, we obtain

$$\begin{aligned} B = & \left[(1+\chi)\, p_1 \left(\varepsilon_1 k + \varepsilon p_2 ctgp_2 h \right) - \chi \varepsilon k_2^2 tgp_1 h \right] \cdot \\ & \left[(1+\chi)\, p_1 \left(\varepsilon_1 k - \varepsilon p_2 ctgp_2 h \right) + \chi \varepsilon k_2^2 tgp_1 h \right]^{-1} A \end{aligned} \quad (31)$$

The numerator and denominator of expression (31), as well as formula (26), vanishes at $p_1 = 0$. After removing the uncertainty, it turns out

$$\begin{aligned} B = & \left[(1+\chi)\left(\varepsilon_1 k + \varepsilon p_2 ctgp_2 h \right) - \chi \varepsilon k_2^2 h \right] \cdot \\ & \cdot \left[(1+\chi)\left(\varepsilon_1 k - \varepsilon p_2 ctgp_2 h \right) + \chi \varepsilon k_2^2 h \right]^{-1} A \end{aligned} \quad (32)$$

CONCLUSION

In the problem of electromagnetic wave incidence on a half-space made of a piezoelectric material of hexagonal symmetry of class 6 mm, the nature of piezoelectric effect influence on transparency condition is established. In the case of reflection from a finite piezolayer, the reflection coefficients are determined for different variants of boundary conditions.

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