ANALYSIS OF EMITTER COUPLED MULTIVIBRATORS

Antonio Buonomo,
Dipartimento di Ingegneria dell'Informazione, Seconda Universita degli Studi di Napoli, Napoli, Italy,
antonio.buonomo@unina2.it

Alessandro Lo Schiavo,
Dipartimento di Ingegneria dell'Informazione, Seconda Universita degli Studi di Napoli, Napoli, Italy,
loschiavo@ieee.org

DOI: 10.36724/2664-066X-2021-7-5-7-11

ABSTRACT

The purpose of this investigation is to develop a systematic nonlinear analysis of the circuit in Figure 1, as yet not available in the literature, obtaining an exact formula for the oscillation frequency. Then, we find the condition for the circuit to behave as a relaxation oscillator, or as a nearly sinusoidal oscillator, analyzing the circuit behavior around the equilibrium point through a circuit model accounting for the transistor intrinsic capacitances. A nonlinear analysis of the popular emitter-coupled multivibrator is performed, which is based on the classical discontinuity theory. Exact relationships for calculating its waveform are obtained. Then, we investigate the circuit dynamic behavior showing that at very high frequencies the circuit exhibits a completely different behavior, similar to an LC second-order nearly sinusoidal oscillator. Conditions allowing us to predict one or the other behavior are found.

KEYWORDS: *Relaxation oscillations, emitter-coupled multivibrator, nearly sinusoidal oscillators, differential VCOs.*

The article is reworked from unpublished 2nd IEEE International Conference on Circuits and Systems for Communications (ICCSC) materials.

INTRODUCTION

The circuit shown in Figure 1 is the well-known emitter-coupled astable multivibrator with a floating timing capacitor [1]. This circuit, and its CMOS companion [2],[3], which can be easily fabricated in monolithic form, is widely used in applications involving waveform generation, such as voltage-controlled frequency, current-controlled oscillators, and I/Q cross-coupled oscillators [4]. The circuit is commonly used as a relaxation oscillator providing a square wave at the collectors of transistors at high frequencies. However, if it is not properly designed, the circuit can exhibit a completely different behavior at very high frequency, giving rise to undesirable nearly sinusoidal oscillations whose origin is not thoroughly elucidated.

The purpose of this investigation is to develop a systematic nonlinear analysis of the circuit in Figure 1, as yet not available in the literature, obtaining an exact formula for the oscillation frequency. Then, we find the condition for the circuit to behave as a relaxation oscillator, or as a nearly sinusoidal oscillator, analyzing the circuit behavior around the equilibrium point through a circuit model accounting for the transistor intrinsic capacitances. This condition enables to elucidate the unusual dynamic behavior of the circuit. Essentially, we show that the circuit, which does not have any apparent inductor, can operate like an LC second-order oscillator, by suitably setting the circuit parameters. This is made possible as the composite two-terminal connected to the timing capacitance, which has an S-type characteristic, behaves like a locally-active inductive two-terminal, i.e. its behavior for small variations around the quiescent operating point is equivalent to a series connection of a negative resistance and an inductance.

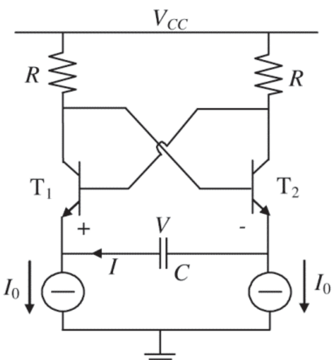

Fig. 1. Schematic of a bipolar multivibrator

Finally, circuit simulations were performed to show that both relaxation and nearly-sinusoidal oscillations can be actually generated.

BEHAVIOR AS RELAXATION OSCILLATOR

As it is known, the condition for the circuit in Figure 1 to behave as an astable multivibrator is that the *I-V* characteristic of the composite two-terminal connected to the capacitance C has a negative slope at its crossing point Q with the axis $I = 0$. In fact, by virtue of the presence of C, this point defines the unique equilibrium configuration of circuit, which is instable if the characteristic slope is negative.

Assuming a purely resistive behavior of the active devices, whose collector currents are expressed by the simplified Ebers-Moll model $I_E = I_S \exp(V_{BE} / V_T)$, from equation $V = R(I_1 - I_2) + (V_{BE2} - V_{BE1})$, we deduce the nonlinear characteristic $V = F(I)$ of the composite two-terminal, which is

$$V = -2RI + V_T \ln\left(\frac{1 + I / I_0}{1 - I / I_0}\right) \quad (1)$$

This characteristic is an odd function which has the typical S-shape shown in Figure 2. To verify that the above condition is met, it is sufficient to ascertain that the derivative of (1) at the origin is negative, which happens if $V_T < RI_0$.

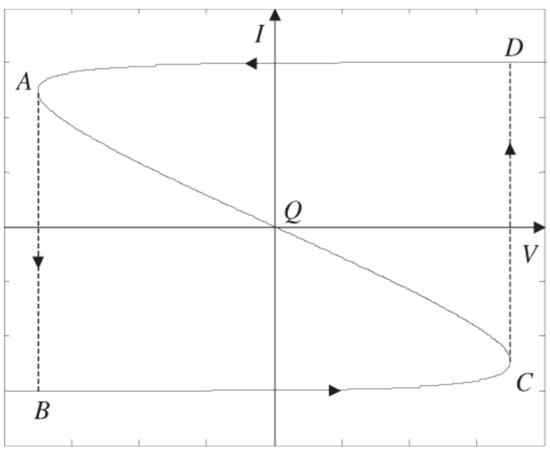

Fig. 2. Static I-V characteristic for the two-terminal connected to the capacitor

The considered circuit is a degenerated self-oscillating system wherein, as it is known, relaxation oscillations take place. Its dynamic behavior is described by the first-order nonlinear differential equation

$$I = -C \frac{d}{dt} F(I) \quad (2)$$

According to the Mandelstam postulate of constant energy [5], we assume the possibility of the existence of infinitely fast jumps of the current I. Accordingly, during the oscillatory process, the system can jump from one steady branch of the curve to the other, i.e. from the point A to point B, and from the point C to the point D. Consequently, the limit cycle is composed by two analytical branches DA and BC, which are closed by the two discontinuities AB and CD. The above assumption simplifies the calculation of the steady-state oscillation, which is here obtained by solving eq. (2).

Before solving (2), we calculate the characteristic points, $A\ (I_c, -V_c)$, $C\ (-I_c\ ,V_c\ )$ (Fig. 2), which are obtained imposing that the derivative dV dI of (1) vanishes. Thus, we get the current value

$$I_c = I_0 \sqrt{1-h} \quad , \qquad h = \frac{V_T}{R\, I_0} \tag{3}$$

and the corresponding voltage

$$V_c = -2\frac{V_T}{h}\sqrt{1-h} + V_T \ln\left(\frac{1+\sqrt{1-h}}{1-\sqrt{1-h}}\right) \tag{4}$$

The relaxation intervals, i.e. the analyticity interval of the current I, are determined solving eq. (2), which can be rearranged in the form

$$dt = \left(2RC\frac{1}{I} - \frac{2\,CI_0\,V_T}{I_0^2 - I^2}\frac{1}{I}\right)dI \tag{5}$$

Integrating this equation between the time instant immediately after a jump, t_0^+, when the current is very close to I_0 (let us call it Ii), and a time instant t, we get

$$t = t_0^+ + 2RC\,(1-h)\ln\left(\frac{I}{I_i}\right) + R\,C\,h\ln\left(\frac{I_0^2 - I^2}{I_0^2 - I_i^2}\right) \tag{6}$$

which provides a relationship between the current and the time, until the other characteristic point is reached. Therefore, we can calculate the time between two consecutive jumps, at t_0^+ and at t_0^+ + T', and, thus, the period of oscillation of the multivibrator as $T = 2T'$. The oscillation frequency results in $f = 1/\ T$, i.e.

$$f = \frac{1}{4RC}\frac{1}{\left(1-\frac{V_T}{RI_0}\right)\ln\left(\frac{I_c}{I_i}\right) + \frac{V_T}{2\,RI_0}\ln\left(\frac{I_0^2 - I_c^2}{I_0^2 - I_i^2}\right)} \tag{7}$$

We conclude observing that the behavior analyzed is characterized by a time-domain waveform of the current presenting two intervals, during which the current varies on the S-shape characteristic from I_i to I_c , and two instantaneous jumps from I_c to I_i . Therefore, the capacitor current and, hence, the collector voltage have a near square waveform.

BEHAVIOR AS NEARLY SINUSOIDAL OSCILLATOR

Now, we show that the circuit in Figure 1 can operate either as a multivibrator, as shown in previous section, or as a nearly-sinusoidal oscillator. In particular, we perform a small-signal analysis around the quiescent operating point Q of circuit showing that it can be unstable and that this instability can be associated with a pair of natural frequencies of the linearized equivalent circuit, which are complex-conjugate with positive real part. Consequently, nearly sinusoidal oscillations can take place in the circuit. To this end, we calculate the impedance seen looking into the emitter terminals of the circuit connected to the capacitor C, which is represented by the equivalent circuit shown in Figure 3.

This circuit is obtained using for the BJTs T_1 and T_2 the simplified small-signal high-frequency model and neglecting the output resistances.

From the circuit in Figure 3, we calculate the impedance seen looking into the emitters

$$Z(s) = k\frac{R_1 + sL_1 + s^2 d}{(1+s\,\tau_\pi)(1+s\,\tau_\mu)} \tag{8}$$

where

$$k = 2/(1+g_m r_\pi)\ ; \tag{9a}$$

$$R_1 = r_b + r_\pi + R - R\ r_\pi\ g_m\ ; \tag{9b}$$

$$d = R\, r_b r_\pi C_\pi C_\mu\ ; \tag{9c}$$

$$L_1 = C_\pi r_\pi (r_b + R) + C_\mu\left[r_\pi (r_b + 4R) + r_b R(1+g_m r_\pi)\right]\ ; \tag{9d}$$

$$\tau_\pi = \frac{C_\pi r_\pi}{(1+g_m r_\pi)}\ ; \quad \tau_\mu = C_\mu (r_b + 4R)\,. \tag{9e}$$

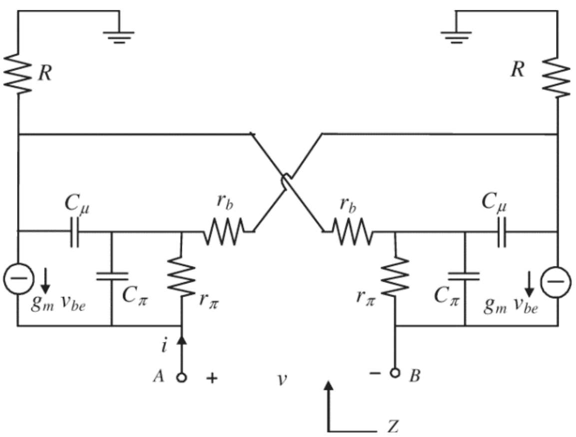

Fig. 3. Small-signal equivalent circuit of the nonlinear two-terminal connected to the capacitor

In order to investigate the behavior around the equilibrium point of the whole circuit in Figure 1, described by the parallel connection of the capacitor and the two-terminal described by the linearized transfer function (8), we analyze the roots of the describing equation

$$Z(s)(-sC) = 1 \tag{10}$$

which can be rewritten as

$$s^3 kdC + s^2 (kL_1C + \tau_\pi \tau_\mu) + s(kR_1C + \tau_\pi + \tau_\mu) + 1 = 0 \tag{11}$$

The roots of equation (11), which can be real or complex, determine different behaviors of the circuit. Actually, if at least one root is positive then the circuit is unstable and an oscillation can take place. Moreover, if all of the roots are real the circuit behaves like a multivibrator, while if two roots are complex-conjugate the circuit behaves like a near-sinusoidal oscillator. The value of the roots and, hence, the actual behavior of the circuit depends on the values assumed by the circuit parameters, as shown in Figure 4, where the roots of (11) are plotted into the s-plane for different values of the transistor parasitic capacitances.

Parameters used are I_0 = 3 mA, β = 50, R = 26Ω, r_b = 160Ω, C_μ = 0,1 C_π, V_{CC} = 3 V, C = 1.5 pF and Cπ varies from 0.01 pF to 10 pF . Therefore, we deduce that increasing parasitic capacitance values make the oscillator behavior to change from relaxation to near-sinusoidal oscillator.

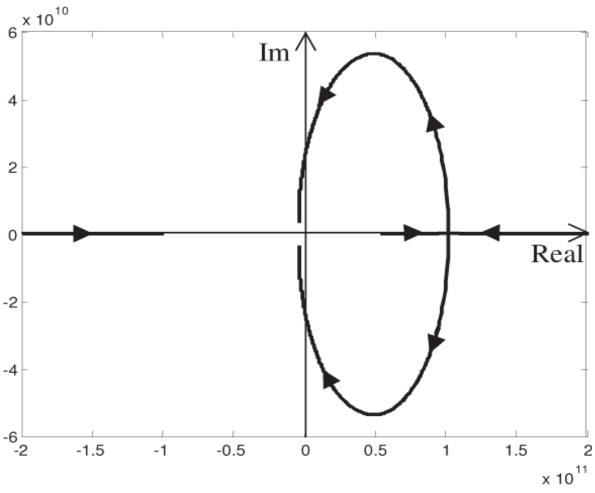

Fig. 4. Root locus for equation (11)

Let us now consider the case that eq.(11) admits a pair of complex-conjugate roots and the expression of $Z(s)$ can be simplified to

$$Z(s) = k\,R_1 + s\,k\,L_1\ . \tag{12}$$

This impedance can be rewritten into the frequency domain in the form

$$Z(\omega) = R_e + j\omega\,L_e \tag{13}$$

with $R_e = kR_1$ and $L_e = kL_1$. Thus, we deduce that the behavior of the cross-coupled transistor pair at the emitter terminals can be equivalent to that of an inductance in series with a resistance, which can be made negative. Consequently, the considered two-terminal can be referred to as locally-active inductive two-terminal, and the whole circuit in Figure 1 can be represented by the series resonant circuit shown in Figure 5.

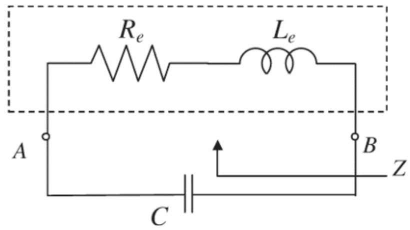

Fig. 5. Equivalent linearized circuit of the multivibrator in Figure 1

Taking into account (12), it can be shown that (10) has a pair of purely imaginary roots, $\pm j\omega_0$, with

$$\omega_0^2 = \frac{1}{LC} \quad ; \qquad L = kL_1 \tag{14}$$

if the following condition holds

$$R_1 = \frac{d}{kL_1C}\ . \tag{15}$$

Moreover, eq. (10) admits a pair of complex-conjugate roots with a positive real part if the following start-up condition

$$R_1 < \frac{d}{kL_1C} \tag{16}$$

is met. Equations (14)-(16) allow us to easily design the circuit in Figure 1 in order to operate as a nearly sinusoidal oscillator. Anyway, it must be highlighted that the steady-state oscillation frequency slightly differs from that shown in (14), due to the effects of nonlinearity [6] and because the circuit parameters are set in order to satisfy (16) and not (15).

When we need to use the complete expression (8) of $Z(s)$, it is easy to show that (11) admits a pair of purely imaginary roots, $\pm j\omega_0$, with

$$\omega_0^2 = \frac{1}{LC} \quad ; \qquad L = kL_1 + \frac{\tau_\pi \tau_\mu}{C} \tag{17}$$

when $R_1 = R_{\lim}$, having defined

$$R_{\lim} = \frac{k\,d\,C\omega_0^2 - \tau_\pi - \tau_\mu}{kC} \quad , \qquad (18)$$

and the start-up condition becomes $R_1 < R_{\lim}$, which relates in a simple way the circuit behavior to circuit parameters, both active and passive.

SIMULATIONS

Numerical simulations of the considered circuit have been performed for different parameter values confirming that it can exhibit different behaviors.

Using the parameters I_0 = 3 mA, β = 50, R = 26Ω, r_b = 160Ω, C_π = 0.1 fF, C_μ = 0,1 C_π, V_{CC} = 3 V and to C = 1.5 pF, the roots of describing equation of the linearized model are real and two of them are positive. Therefore, the circuit is unstable and behaves like a relaxation oscillator, as shown by the SPICE transient simulation in Figure 6.

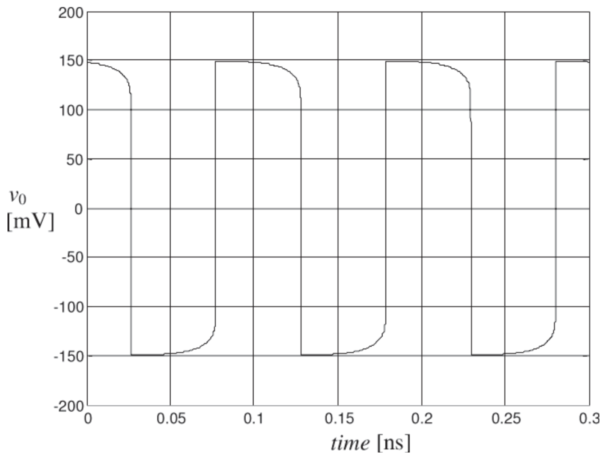

Fig. 6. Time-domain waveform produced by the circuit operating as a relaxation oscillator

On the other hand, using the same circuit parameters as above, just increasing the value of the parasitic capacitances, i.e. $C\pi$ = 1 pF and C_μ = 0,1 pF, the linearized model admits a pair of complex-conjugate roots with positive real part.

In this case the equivalent circuit in Figure 5 predicts an interaction between the equivalent inductance and the external capacitance, leading to a near sinusoidal oscillation. As the start-up condition is satisfied, a near sinusoidal oscillation is observed between the collectors (v_0), as shown by the SPICE transient simulation reported in Figure 7.

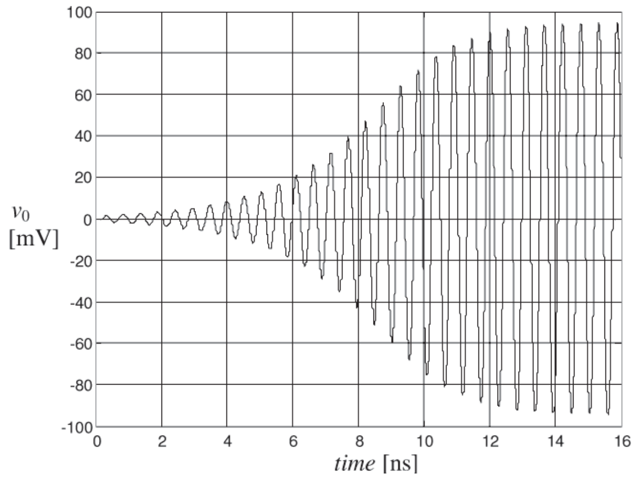

Fig. 7. Time-domain waveform produced by the circuit operating as a nearly sinusoidal oscillator

CONCLUSIONS

We presented a nonlinear analysis of the emitter coupled multivibrator, obtaining exact relationships for calculating its waveform. Then, we showed that the circuit can operate as an LC-tuned nearly-sinusoidal oscillator by suitably setting the circuit parameters. In this case, the circuit represents an alternative realization of a differential VCO which does not require an external or integrated inductor.

REFERENCES

[1] A.A. Abidi and R.G. Meyer. Noise in relaxation oscillators, *IEEE J. Solid-State Circuits*, vol. 18, pp. 794-802, Dec. 1983.

[2] I.M. Filanovsky. Remarks on design of emitter- coupled multivibrators, *IEEE Trans. on Circuits and Syst.*, vol. 35, pp. 751-755, June 1988.

[3] B.J. Song, H. Kim, Y. Choi and W. Kim. A 50% power reduction scheme for Cmos relaxation oscillator, *AP-ASIC ’99*, pp. 154-157, 1999.

[4] J.R. Fernandes, M.H.L. Kouwehoven, C. van der Bos, K. van Hartingsveldt and C. Verhoeven. A 5.8 Ghz quadrature cross-coupled oscillator, *Proc. IEEE ISCAS*, vol. II, pp. 404-407, 2002.

[5] A.A. Andronov, A.A. Vitt and S.E. Khaikin, Theory of oscillators, Pergamon Press, 1966.

[6] Buonomo and A. Lo Schiavo. Analyzing the Dynamic Behavior of RF Oscillators, *IEEE Trans. on Circuits and Syst.-Part I,* vol. 49, pp. 1525-1534, 2002.