# PERFORMANCE ANALYSIS OF ASYNCHRONOUS FFH-MA UNDER THE PRESENCE OF THE FREQUENCY OFFSET

**Jeungmin Joo, Kanghee Kim, Hyunduk Kang, and Kiseon Kim,**
*Department of Information and Communications,*
*Kwangju Institute of Science and Technology, Kwangju, Republic of Korea,*
*gangsang@kjist.ac.kr*

DOI: 10.36724/2664-066X-2021-7-5-26-30

**ABSTRACT**

In order to investigate the effect of asynchronous fast frequency hopping-multiple access (FFH-MA) systems due to the frequency offset, we evaluate the bit error rate (BER) performance, using noncoherent *M*-ary frequency shift keying (FSK) modulation in the Rayleigh fading channel. While the frequency offset increases at a given signal to noise ratio (SNR), the BER is severely degraded due to the loss of orthogonality of received symbols. With 10% frequency offset, about 5 dB SNR is required additionally to obtain $2 \times 10^{-3}$ BER, compared to that in the perfectly frequency synchronized case. For the SNR of more than 20 dB, the threshold level of the receiver suffering from the frequency offsets should be greater than that of the perfectly synchronized receiver.

**KEYWORDS:** *Hysteretic Oscillator, Frequency domain techniques, Floquet's multipliers.*

*The article is reworked from unpublished 2nd IEEE International Conference on Circuits and Systems for Communications (ICCSC) materials.*

## Introduction

For several decades, frequency hopping-multiple access (FH-MA) communication systems have become an attractive technique. Both of synchronous and asynchronous FH-MA systems have been used extensively in the areas of satellite and ground military applications. Recently, FH-MA systems have become more common for commercial applications in the license-free industrial, scientific, and medical (ISM) bands [1].

The performance analysis of the various synchronous FH-MA systems have been evaluated in [2]-[5]. In the asynchronous case, however, it is not easy to analyze the system performance due to the difficulties in handling asynchronocity of the multiple interferers. The evaluation of multiple integral to obtain the expression for the probability of error, which is not very attractive, is required. Many literature works related to asynchronous FH-MA systems have been reported. At first, the error probability is upper bounded by 1/2 whenever a hop is hit by interferers in [6]. In [7] and [8], a Gaussian approximation method was used to simply approximate the probability error. In [9], the effect of non-orthogonality of the interfering signals due to the asynchronocity (i.e., random delays) of users was neglected in the analysis on the assumption that the frequency separation between two symbols is much larger than the minimum required to guarantee orthogonality. In [10] and [11], expression for the probability of error was derived when a hop is hit only by one interfering user. Recently, in [12], by using characteristic function and two dimensional integral, the approximated error probability was obtained.

Like any other digital communication systems, the FH-MA system suffers from frequency offsets caused by the imperfect estimate of hopping frequencies in the dehopper. Hence, it is interesting to investigate the effect of frequency offset in asynchronous FH-MA systems.

In this paper, we analyze bit error probability of the fast FH-MA (FFH-MA) system considering all possible hits under the presence of frequency offset.

This paper is organized as follows: Section II describes the system and channel models. In Section III, the BER formulas for the FFH-MA system are derived under the presence of the frequency offset. Numerical results are described in Section IV, and conclusions are given in Section V.

## System and Channel Model

### A. Transmitter Model

The binary data sequence of rate $R_b$ is mapped into an $M$-ary frequency shift keying (MFSK) signal sequence of rate $R_s$, where $R_s = 1/T_s = R_b/\log_2 M$. Let the $k$th user transmit a symbol, $m_k$, for $T_s$, where $m_k \in \{0, 1, \cdots, M-1\}$. The symbol representing an MFSK signal is converted to a constant $L$-tuple representing a $\log_2 M$ bits message, i.e.,$\boldsymbol{m}_k = \{m_k, m_k, \cdots, m_k\}$. The message represented as an $L$-tuple is added modulo-$M$ to an address sequence which is an $L$-tuple and unique for each user, i.e., $\boldsymbol{a}_k = \{a_{k0}, a_{k1}, \cdots, a_{k(L-1)}\}$, where $a_i \in \{0, 1, \cdots, M-1\}$. As a result, it produces a new sequence of length $L$, i.e., $\boldsymbol{b}_k = \{b_{k0}, b_{k1}, \cdots, b_{k(L-1)}\} = \boldsymbol{m}_k \oplus \boldsymbol{a}_k$, where $\oplus$ denotes modulo-$M$ addition. Each codeword, $b_{kl}$, is used for selecting one out of the available $M$ orthogonal frequencies. Each codeword modulated by the MFSK modulator is then hopped to one among $Q$ hopping frequencies per hop duration, $T_h = T_s/L$. Each user is assigned independently a Markov hopping pattern which makes two consecutive hopping frequencies to be different. The signal transmitted by the $k$th user for $T_s$ is given by

$$S_k(t) = \sum_{l=0}^{L-1} \sqrt{2P} p_{T_h}(t - lT_h) \exp[j(2\pi(f_c + f_{kl})t + \theta_{kl})], \quad (1)$$

where $p_{T_h}(t)$ is a unit amplitude pulse of the hop duration. $P$ and $f_c$ are the transmitted power and carrier frequency, respectively, and $\theta_{kl}$ is random phase introduced by the frequency hopper. $f_{kl} = q_{kl}M/T_h + b_{kl}/T_h$, where $q_{kl} \in \{0, 1, \cdots, Q-1\}$ is a hopping frequency index.

Let $K$ active users exist in the FFH-MA system. According to the frequency hopping pattern and address sequence of the desired user, the received signal is dehopped. When $J_l$ interferers cause hits for the $l$th hop, the resulting output signal is represented as

$$r_d(t) = \sum_{l=0}^{L-1} \sum_{k=0}^{J_l} \alpha_{kl} \exp(j\phi_{kl}) p_{T_h}(t - (l - \varepsilon_k)T_h) \exp[j(2\pi(f_c + g_{kl}/T_h + \Delta f)t + \theta_{kl})] + n(t), \quad (2)$$

where the part corresponding to $k = 0$ is the desired user's signal, and $g_{kl} = (b_{kl} - a_{0l}) \bmod M$. Note that $g_{0l} = m_0$ for all $l$. $\Delta f$ denotes the frequency offset. It is assumed that the frequency offset, $\Delta f$, is the same for $L$ hops of all users, and less than the minimum frequency spacing among $M$-ary symbols, i.e., $-1/T_h \leq \Delta f \leq 1/T_h$. $\alpha_{kl}$ is the channel gain having Rayleigh distribution, $\phi_{kl}$ is the phase of channel having uniform distribution $[0, 2\pi)$, and $n(t)$ denotes the AWGN with two-sided power spectral density of $N_0/2$. Note that uncertainty about phase of the received signal, $\theta_{kl} + \phi_{kl}$, is also uniformly distributed within $[0, 2\pi)$, and independent of $\alpha_{kl}$ [12]. $\varepsilon_k$ is timing misalignment relative to the desired signal, where $\varepsilon_k$ is uniformly distributed within $[0, 1)$ for $k \neq 0$ and $\varepsilon_0$ is set to be zero.

The dehopped signal passes through the noncoherent MFSK demodulator, which consists of $M$ envelope detectors. The output of the $n$th envelope detector is represented as

$$R_{nl} = \left| \frac{2}{T_h} \int_{lT_h}^{(l+1)T_h} r_d(t) \exp(j2\pi(f_c + n/T_h)t)dt \right|, \quad (3)$$

where $|\cdot|$ means an absolute value of the argument. We assume that the average received power per hop is the same for all $L$ hops. Then, the conditional probability density function (pdf) of $R_{nl}$ has the Rayleigh distribution given by

$$f_{R_{nl}}(R|\Omega^2) = \frac{R}{\Omega^2} \exp\left(-\frac{R^2}{2\Omega^2}\right), \quad (4)$$

where

$$\Omega^2 = \sum_{k=0}^{J_l} P_{ave} \operatorname{sinc}^2(n_{kl} + \rho) + \sum_{k=1}^{J_l} \Big\{ P_{ave}(1 - \varepsilon_k)^2 \cdot \operatorname{sinc}^2((n_{kl} + \rho)(1 - \varepsilon_k)) \Big\} + N_0/T_h, \quad (5)$$

where $\operatorname{sinc}(x) \stackrel{\text{def}}{=} \sin(\pi x)/\pi x$, $P_{ave}$ is the average received power, $\rho$ is normalized frequency offset, i.e., $\rho = \Delta f / T_h$, and $n_{kl} = g_{kl} - n$. Note that even though interferers' signals are received asynchronously, $R_{nl}$ does not contain interferences caused by adjacent symbols around the $l$th symbol. The reason is why the Markov hopping pattern is used.

The demodulated signal passes through hard-decision detector (the resulting output is 1 or 0), and the receiver's time-frequency (T-F) matrix can be reconstructed [3, 12].

## BER Performance Analysis

By using (4), the insertion probability is obtained, which is used for the hard-decision decoding and majority logic decision. With the majority logic decision, it is investigated that how many entries exist in spurious rows and a desired row of the T-F matrix. Then, based on the majority logic decision rule, if a desired row contains more entries than any spurious rows, a correct decision is attainable, otherwise, error will occur [3].

### A. Insertion Probability

The insertion probability is defined as the probability that the output of the envelope detector is larger in magnitude than a given threshold, $\beta$. Let $\boldsymbol{n}_l = \{n_{0l}, n_{1l}, \cdots, n_{J_l l}\}$ and $\boldsymbol{\varepsilon} = \{\varepsilon_0, \varepsilon_1, \cdots, \varepsilon_{J_l}\}$. Conditioned on $\boldsymbol{n}_l$ and $\boldsymbol{\varepsilon}$, the insertion probability at the $n$th envelope detector for the $l$th hop can be given by

$$P_{Inl}(\boldsymbol{n}_l, \boldsymbol{\varepsilon}_l) = \int_0^\beta f_{R_{nl}}(R|\Omega^2)dR = \exp\left(\frac{-B}{\chi(\boldsymbol{n}_l, \boldsymbol{\varepsilon})}\right), \quad (6)$$

where $B = \beta_0^2 N_0 / 2\bar{E}_h$, and

$$\chi(\boldsymbol{n}_l, \boldsymbol{\varepsilon}) = \sum_{k=1}^{J_l} \chi_k(n_{kl}, \varepsilon_k) = \sum_{k=1}^{J_l} [\operatorname{sinc}^2(n_{0l} + \rho) + N_0/\bar{E}_h]/J + (1 - \varepsilon_k)^2 \operatorname{sinc}^2[(n_{kl} + \rho)(1 - \varepsilon_k)], \quad (7)$$

where $\bar{E}_h/N_0$ is the average received SNR per hop, and $\beta_0 = \beta/\sqrt{N_0/T_h}$ is the actual threshold level $\beta$ normalized by the rms noise power, which is used commonly in envelope detectors. This threshold level needs to be selected properly to optimize the BER performance. The selection of a threshold level depends on system parameters and channel conditions. (6) contains $J_l$ random variables related to the asynchronous transmission. For simplifying the analysis, the characteristic function can be used. Let $\chi(\boldsymbol{n}_l, \boldsymbol{\varepsilon}) = \chi_l$ and $\chi_k(n_{kl}, \varepsilon_k) = \chi_{kl}$ for simplifying the notation. When $f_{\chi_l}(x|\boldsymbol{n}_l)$ exists over $x_L \leq x \leq x_U$, (6) can be represented as

$$P_{Inl}(\boldsymbol{n}_l) = \int_{x_L}^{x_U} \exp(-B/x) f_{\chi_l}(x|\boldsymbol{n}_l)dx = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Phi_{\chi_l}(\omega|\boldsymbol{n}_l)H(\omega)d\omega, \quad (8)$$

where $\Phi_{\chi_l}(\omega|\boldsymbol{n}_l)$ is the characteristic function of $f_{\chi_l}(x|\boldsymbol{n}_l)$, which is defined as $\int f_{\chi_l}(x|n_l)\exp(j\omega x)dx$, and

$$H(\omega) = \sum_{k=1}^{\infty} \frac{(-B)^k}{k!} \left\{ \sum_{n=1}^{k-1} \left[ \frac{(-j\omega)^{n-1}}{(k-1)\cdots(k-n)} \frac{\exp(-j\omega x)}{x^{k-n}} \right] + \frac{(-j\omega)^{k-1}}{(k-1)!} E_i(-j\omega x) \right\} + \frac{j}{\omega} \exp(-j\omega x) \Bigg|_{x=x_L}^{x_U}, \quad (9)$$

where $j \stackrel{\text{def}}{=} \sqrt{-1}$ and $f(x)|_a^b$ denotes $f(b) - f(a)$. $E_i(\pm jx) = \operatorname{Ci}(x) \pm j\operatorname{Si}(x)$, where $\operatorname{Ci}(x) = -\int_x^\infty \frac{cos(t)}{t}dt$ and $\operatorname{Si}(x) = -\frac{\pi}{2} + \int_0^x \frac{sin(t)}{t}dt$ are called Cosine integral and Sine integral, respectively [13].

Since $\{\chi_{kl} | k = 1, 2, \cdots, J_l\}$ are independent random variables, $\Phi_{\chi_l}(\omega|\boldsymbol{n}_l) = \prod_{k=1}^{J} \Phi_{\chi_{kl}}(\omega|n_{kl})$. Conditioned on $n_{kl}$, the pdf of $\chi_{kl}$ is given by

$$f_{\chi_{kl}}(x|n_{kl}) = \begin{cases} \frac{[2|n_{kl}|+\mathrm{sgn}(n_{kl})]/|n_{kl}+\rho|}{2\pi\sqrt{\frac{x-a}{[\pi(n_{kl}-\rho)]^2}-(x-a)^2}} & \text{for } a \le x \le b \\ \frac{|n_{kl}|/(\pi|n_{kl}+\rho|)}{\sqrt{\frac{x-a}{[\pi(n_{kl}-\rho)]^2}-(x-a)^2}} & \text{for } b < x \le c, \end{cases} \quad (10)$$

where $a = [\mathrm{sinc}^2(n_{0l}+\rho) + N_0/\bar{E}_h]/J$, $b = a + \frac{1-\cos(2\pi\rho)}{2(\pi n_{kl})^2}$, and $c = a + \frac{1}{(\pi n_{kl})^2}$. $\mathrm{sgn}(x) = 1$ for $x \ge 0$ and $\mathrm{sgn}(x) = -1$, otherwise. By using Taylor series ($\frac{1}{\sqrt{1+x}} = 1 - \frac{1}{2}x + \frac{3}{8}x^2 - \cdots$) and some manipulations, $\Phi_{\chi_{kl}}(\omega|n_{kl})$ can be given by

$$\Phi_{\chi_{jl}}(\omega|n_{jl}) = \frac{2|n_{jl}| + \mathrm{sgn}(n_{jl})}{2\exp(-j\omega a)} \int_0^{b-a} F(t)e^{j\omega t}dt + |n_{jl}|\exp(j\omega a)\int_{b-a}^{c-a} F(t)e^{j\omega t}dt, \quad (11)$$

where $F(t) = \frac{1}{\sqrt{t - \frac{t^2}{c-a}}} = \sum_{n=0}^{\infty}\left(\prod_{k=0}^{n}\frac{1+2k}{2k+\delta(k)}\right)\frac{t^{-0.5+n}}{(c-a)^n}$ and $\delta(k)$ is the kronecker delta function. (11) can be simplified with the following relationships [13]:

$$\int x^t e^{zx}dx = \frac{x^t e^{zx}}{z} - \frac{t}{z}\int x^{t-1}e^{zx}dx \quad (12)$$

$$\int \frac{e^{zx}}{\sqrt{x}}dx = \sqrt{\frac{\pi}{-z}}\mathrm{erf}(\sqrt{-zx}), \quad (13)$$

where $\mathrm{erf}(x) \stackrel{\text{def}}{=} \frac{2}{\pi}\int_0^x e^{-t^2}dt$. (12) and (13) can be used for evaluating (11) by letting $t = \{-0.5 + n|n = 0, 1, 2, 3, \cdots\}$ and $z = j\omega$. However, it still requires infinite summation to get the exact result of (11) when $F(t)$ is converted to Taylor series. Numerical observation shows that when $F(t)$ is represented as Taylor series, $F(t)$ specified within $0 \le t \le (c-a)/2$ can be represented as the summation of at most 10 terms. However, in order to represent $F(t)$ specified within $(c-a)/2 < t \le (c-a)$, the summation of at least 100 terms is required. In other words, the amount of summation depends on the integral interval, or $\rho$ in (11). To solve this problem, we use the symmetric property of $F(t)$ for $0 < t < (c-a)$ . For example, for $t_2 > t_1 \ge (c-a)/2$, $\int_{t_1}^{t_2} e^{j\omega t}F(t)dt = e^{j\omega(c-a)}\int_{c-a-t_2}^{c-a-t_1} e^{-j\omega t}F(t)dt$. When using above relationship, it is enough to use at most 10 terms of $F(t)$ represented as Tayler series.

### B. Bit Error Rate

Due to asynchronous transmission and frequency offset, different received MFSK symbols are no longer orthogonal. On the assumption that the $m$th symbol has been transmitted by the desired user, the insertion probability at each envelope detector is averaged over all transmission symbols of interferers.

$$P_{Inl}(m) = \frac{1}{M^J}\sum_{\substack{\text{all } n_{kl} \\ \text{except } n_{0l}=m-n}} P_{Inl}(\boldsymbol{n}_l) \quad (14)$$

where $\boldsymbol{n}_l = \{n_{0l}, n_{1l}, \cdots, n_{Jl}\}$, $n_{kl} = g_{kl} - n$, and $g_{kl} \in \{0, 1, \cdots, M-1\}$. Then, the insertion probabilities of entries at the desired row and spurious rows are given by $P_{Iml}(m)$ and $P_{Inl}(m)$, respectively, where $n \ne m$.

The probability that there are $i$ entries in the $n$th row is given by

$$P_n(i|m) = \sum_{\substack{l_1,\cdots,l_i \\ 0\le l_1<\cdots<l_i\le L-1}} \prod_{x=1}^{i} P_{Inl_x}(m) \prod_{\substack{\text{all } l\in\{0,\cdots,L-1\} \\ l\ne l_1\ne\cdots\ne l_i}} (1-P_{Inl}(m)) \quad (15)$$

The probability that $i$ is the maximum number of entries and exactly $k$ spurious rows contain $i$ entries is given by

$$P(i,k|m) = \sum_{\substack{n_1,\cdots,n_k \\ 0\le n_1<\cdots<n_k\le M-1}} \prod_{x=1}^{k} P_{n_x}(i|m) \prod_{\substack{\text{all } n\in\{0,\cdots,M-1\} \\ n\ne n_1\ne\cdots\ne n_k}} \sum_{k=0}^{i-1} P_n(k|m) \quad (16)$$

Let $\boldsymbol{J} = \{J_0, J_1, \cdots, J_{L-1}\}$ be a given hit pattern for $L$ hops. Conditioned on a given hit pattern, the conditional probability of correct symbol detection is given by

$$P_c(\boldsymbol{J}) = \frac{1}{M}\sum_{m=0}^{M-1}\sum_{i=0}^{L} P_m(i|m)\sum_{k=0}^{M-1}\frac{1}{k+1}P(i,k|m) \quad (17)$$

Then, the probability of correct symbol detection is given by

$$P_c = \sum_{J_0,J_2,\cdots,J_{L-1},} P_c(\boldsymbol{J})\prod_{l=0}^{L-1} P_h(J_l) \quad (18)$$

where the probability that $J_l$ of total $(K-1)$ interferers cause hits for the $l$th hop is given by $P_h(J_l) = \binom{K-1}{J_l}(\frac{2}{Q})^{J_l}(1-\frac{2}{Q})^{K-J_l-1}$ [13]. Finally, the BER is obtained by $P_b = \frac{M}{2(M-1)}(1-P_c)$

## Numerical Results

For the numerical analysis, it is assumed that 4 active users exist in the system, and the separation between adjacent hopping frequencies is assumed to be larger than the coherent bandwidth so that each hopping frequency channel fades independently.

In Fig. 1, the sensitivity of the BER performance in the asynchronous FFH-MA system due to the frequency offset is illustrated using the 4-ary FSK signaling. The frequency offset and asynchronous transmission of users destroy the orthogonality among received symbols, giving rise to significant interference. The increase of the frequency offset induces the increase of loss of the desired signal's energy, so that the BER would be more degraded. For getting the BER of $10^{-2}$ at given normalized frequency offset of $\rho$=0.2, the SNR of about 1.5dB is additionally required, compared to the case of the perfect frequency synchronization. With the normalized frequency offset of more than 0.2, however, the BER of $10^{-2}$ is not obtainable even though the SNR increases. Moreover, with the normalized frequency offset of more than 0.4, it is difficult to achieve the improvement of the BER performance simply by increasing the SNR.

Figure 2 shows normalized threshold levels commonly used in envelope detectors to obtain the BER performance depicted in Fig. 1. A numerical search algorithm is employed for specific values of the SNR and frequency offset to obtain normalized threshold values optimizing the BER performance. The step size for the search algorithm is set to 0.1, because it is small enough to optimize the BER. Such normalized threshold levels depend on given system parameters (i.e., $M$, $Q$, and $L$) and channel conditions. Figure 2 shows that, with the existence of the frequency offset, the normalized threshold level for the large SNR of more than about 20dB is quite high compared to the case of the perfect frequency synchronization (i.e., $\rho$=0). This implies that when the SNR increases, the amount that the desired user's signal loses its own energy due to the frequency offset becomes larger than the amount that interferers' signals are robbed of their own energy by the desired envelope detector, because most of energy that interferers' signals lose is absorbed into unwanted envelope detectors. Consequently, if the threshold level for the perfect frequency synchronization system is used for the system suffering from the frequency offset, the BER performance of the system with the frequency offset would be more degraded.

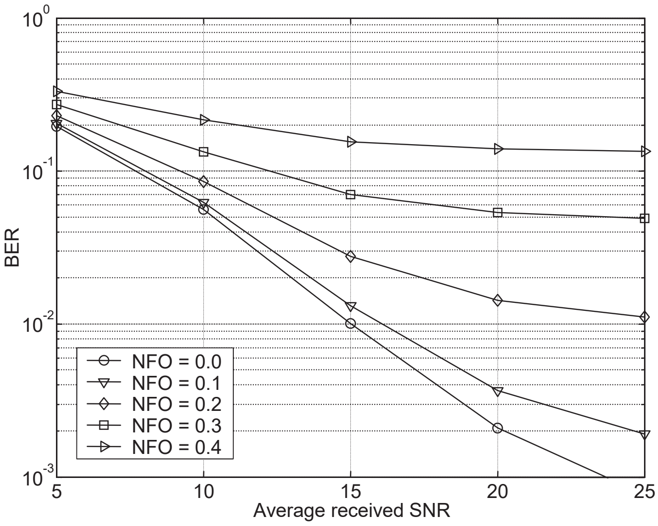


**Fig. 1.** Sensitivity of the asynchronous FFH-MA system due to frequency offsets for $M = 4$, $Q = 55$, $L = 3$, $N_s/N_c = 1/3$, where NFO means the normalized frequency offset

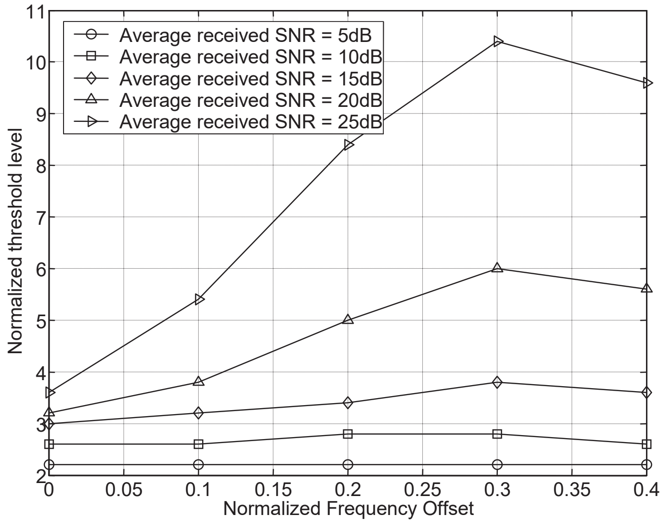


**Fig. 2.** Normalized threshold levels used in Figure 1

## Conclusions

Frequency offsets give rise to the reduction of the desired signal's energy and cause the interference among symbols. The BER performance is severely degraded with the increase of the frequency offsets. For large SNR values of more than 20dB, as the frequency offset increases, the normalized threshold level also increases compared to that used for the perfect frequency synchronization.

## Acknowledgment

This work is supported in part by BK-21 program and ITRC support Program.

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