TOPICS IN COMMUNICATIONS WITH CHAOTIC SYSTEMS

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ABSTRACT

The use of chaotic systems in communications approaches the threshold for industrial applicability. We shall give here an overview over a few techniques we think are promising. However, this overview does not pretend to be complete and it contains personal views on the subject. Also, the use of chaotic system for information encryption is not discussed at all. Only a choice of methods for coding and modulation for the transmission of digital information will be presented. An overview of a number of techniques to transmit information using chaotic systems is given. The difficulties in obtaining a good performance of such systems with respect to channel noise leads to fundamental question, to which we give a possible answer. Given the generally mediocre performance of such approaches, we have identified two fundamental questions which should help us pinpoint the main problems with the existing approaches with the help of information theory and communication theory. In particular it has been confirmed both in theory and by a constructive approach leading to a controlled variant of CSK, that indeed the information generated by free running chaotic systems (as captured by the Kolmogorov-Sinai entropy) is the problem, and the solution is a suitable control that makes this information part of the payload.

KEYWORDS: *Chaos, Communications, Noise performance, information theory, modulation, coding.*

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Introduction

The main features of chaotic systems that can be advantageously exploited for digital communications are

- Chaotic systems are simple to implement, at least at low frequencies [1]. Analog or mixed signal circuits can be used.
- Chaotic signals, i.e. signals produced by chaotic systems can be designed to have a smooth spectrum. The shape of the spectrum can be engineered [2, 3].

Direct chaotic communications

In direct chaotic communications [4], the chaotic signal that carries the information is directly sent over the communication channel. Since chaotic signals have a relatively wide spectrum, this method can only be applied in Ultra-Wideband communications. The basic block diagram of such a communication system is very simple (Fig. 1).

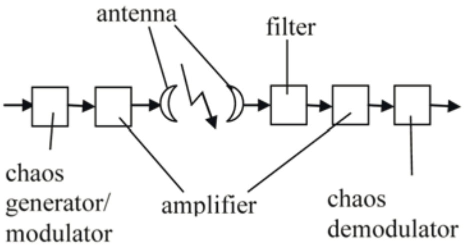

Fig. 1. Block diagram for direct chaotic communications

The advantage of this scheme is its simplicity and its low power consumption, by avoiding up- and down-modulation to base- or intermediate frequency bands. The difficulty lies in the realization of a chaos generator of sufficient quality. Related to this is the limitation to relatively simple modulation schemes, such as on-off keying. With respect to more conventional Ultra-Wideband communication schemes, its advantage lies in a smooth power spectrum of the emitted signal and thus lower peaks of its power spectrum for given total power.

Spreading codes using chaotic systems

In direct-sequence spread spectrum communications, for each bit that is to be transmitted, a rather long binary codeword is sent. For a whole message, the same codeword is used, multiplied either by 1 or by -1, depending on the value of the bit. This procedure increases the bandwidth of the transmitted signal. For this reason, the term spread spectrum is used and the codeword is called spreading sequence. The exaggerated use of frequency spectrum by a single user is compensated by letting several users with close to orthogonal spreading codes share the same frequency band.

Usually, special pseudorandom sequences, the Gold or M-codes, are used as spreading sequences. By using discrete-time chaotic systems whose output signal is 1-bit quantized, a large number of other pseudo-random codes can be generated. Usually, iterations of 1-dimensional maps are used to generate the spreading sequences. Careful optimization of the 1-dimensional maps allow to improve performance with respect to interference with other users of the same frequency band [5] (who use different spreading sequences), as well as multi-path interference performance [6].

In this application, no chaotic signal is sent over the communication channel, chaotic systems are only used to produce finite-length binary signals that have properties suitable for serving as spreading sequences in DS-CDMA systems. The merits and drawbacks of DS-CDMA systems are not any different than when conventional spreading sequences are used, except that more efficient spreading sequences can be produced. In addition, the number of good spreading sequences at the disposal of the system designer is increased considerably by using chaotic systems.

Chaos shift keying (CSK) And differential chaos shift keying (DCSK)

This techniques have been proposed already a decade ago [7, 8, 9, 10] and since then their performance with respect to additive white Gaussian noise in the channel (AWGN-channel) has been studied in depth [11, 12, 13].

This class of techniques mixes the information with a chaotic signal in the base-band or in an intermediate band, and then up-modulates them with a sinusoidal carrier signal. On the receiver side, the signal is first down-modulated before the information is extracted from the chaotic signal (Fig. 2).

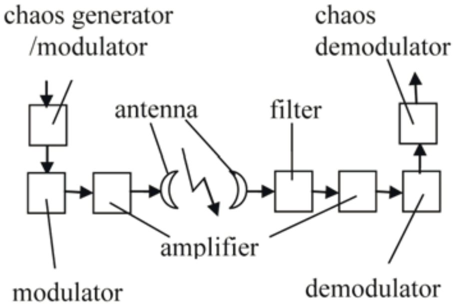

Fig. 2. Block diagram for CSK and DCSK

In principle, CSK and DCSK could also be applied for direct communications and therefore avoiding the implementation of power-consuming additional modulators and demodulators, but given the high frequencies involved, this is out of the reach of current technology. A prototype system on the basis of frequency modulated DCSK (FM-DCSK) has been built and successfully tested [14].

The advantages of using chaos for this class of methods are:

- Relatively simple implementation and thus potentially also a lower power consumption.
- Good robustness properties with respect to interferences

It must be admitted, though, that the second point has in general (beyond additive white gaussian channel noise) not been seriously addressed in the research literature, except for DCSK in [15]. The reason is that the performance for AWG noisy channels is not sufficiently good to motivate communication systems engineers to cross the threshold and invest in understanding techniques based on chaos. In fact, so far, to get close to the performance levels of the basic phase shift keying (PSK) or quadrature amplitude modulation (QAM) already is considered an achievement in the chaos communications community.

Help from information theory

We have wondered whether a better understanding of the situation could be provided by information theory. We were asking whether

- The mediocre performance of CSK, and to a minor extent also DCSK, could be understood by information theoretic arguments
- Whether chaotic signals by their very nature are a bad choice for good performance in communicating over AWGN channels.

Today, we believe that the answer to the first question is “yes” and to the second question “no”. This shall be explained in some more detail.

The principle of CSK is schematically represented in Figure 3.

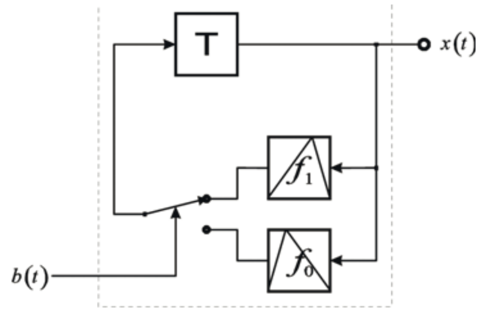

Fig. 3. Block diagram of a chaos shift keying (CSK) transmitter

Depending on the transmitted bit at time t (0 or 1), the transmitter sends a certain number of iterations of the function f_0 or f_1 over the noisy channel. This signal, corresponding to a single bit, is the time-discrete but value-continuous spreading sequence. Note that, unlike conventional DS-CDMA, not only the spreading sequence is not quantized, but it also never repeats. This can be considered an advantage, because the power spectrum of the sent signal has no peak at the frequency at which the bit is transmitted. However, it makes the decoding much more difficult.

At the receiver, it has to be decided, whether the noise-corrupted spreading sequence has been produced by f0 or f1. In the early days of CSK, this was done by chaos synchronization [7, 8, 9].

Unfortunately, straight-forward synchronization was too sensitive to noise perturbation in the channel, and more sophisticated demodulation/decoding methods had to be used [11]. Indeed, this improved the noise-efficiency of the method greatly, but not enough to motivate “communication engineers not already committed” to chaos-communications.

Paper [16] gives the answer. It reminds the readers the well known fact that the chaotic signal, due to its expansive nature that amplifies differences in initial conditions, intrinsically produces at each iteration information about the initial condition of the spreading sequence. In CSK, this unused information has to share the channel capacity with the information that interests us, the use of the functions f0 or f1 for each iteration. Therefore, in this context only two approaches can, in principle, get a noise efficiency close to Shannon’s limit, namely to code information on the initial conditions, or to prevent the information about the initial condition to transit on the channel. The second approach will be discussed in the next section.

In paper [11] a clear hint to the fact that chaotic signals can be used for channel coding is given and that nothing of fundamental nature prevents them from achieving Shannon’s limit for AWGN channels. In order to prove this conclusively, one would have to give an explicit block channel coding method, which in the limit of infinite codeword length reaches the performance of Shannon’s capacity, as is been done in the classical proof [17] using stochastic signals. Of course, such a channel coding method would be too complex to be used for any practical purpose, but it definitely would answer the second question posed above. In paper [11], actually decoding is done using only a simple threshold decision. It would be practical, but it does not achieve Shannon’s capacity. Nevertheless it is orders of magnitudes wrong which makes us believe that the answer to question 2 should be “no”.

Coded modulation using chaotic systems

We now return to the idea to code information onto the initial condition of the spreading code, or, more generally, on the initial conditions of the chaotic signal sent over the channel, not specifying at this point the method the modulation method.

There is a powerful mathematical theory for chaotic systems, or at least for a substantial subset of them: Symbolic analysis [18]. It associates with a chaotic trajectory, usually but not necessarily in discrete time, a binary signal. This correspondence is usually straight forward, just divide the state space into a two regions, label them with 0 and 1, and associate at each time with the state of the system the label of the region. This immediately gives a method to code binary information onto a chaotic trajectory.

Just send the chaotic signal over the channel whose symbolic signal is the information to be transmitted. The problem with this procedure is that one would have to set the initial conditions of the trajectory with unrealistic precision. The way to avoid this is to control the trajectory with small control inputs. This method of communications using chaos has already been proposed a decade ago [19, 20]. Also, such small perturbation control has been found to be feasible for several "more physical" chaotic oscillators such as the Lorenz system [24].

As a result, we can code the payload information directly onto the symbolic sequence generated by the chaotic system (which is equivalent to assigning different initial conditions to different codewords). While in [11] we have demonstrated the feasibility of such an approach in principle, a constructive solution to the problem requires a careful choice of both the chaotic system (in particular the associated nonlinearity) as well as the set of initial conditions representing the codewords.

One possible class of chaotic systems for which the problem of both system design and codeword assignment can be solved in a relatively systematic way is the class of iterated piecewise linear Markov maps. Here, the notion of symbolic dynamics is particularly intuitive, as the symbols can be assigned directly to the intervals of the map and it can be shown that such assignment is "sufficient" in the sense that it covers the entire information production of the chaotic system in the sense of a Shannon and Kolmogorov-Sinai entropy.

With a control action given by $x(t+1)=f(x(t)) + b(t)\cdot q/2$ (for suitable maps $f(.)$ such as the Bernoulli shift map or the tent map, and for suitable coupling coefficients $q=2\text{-}Q$), there is in fact a discrete set of $2Q$ invariant points under this iteration, which allows to control the symbolic sequence [18] with a delay of Q iterations. It has been shown [21] that such a structure can be equivalently represented by a shift register structure with a mapping function generating the output $x(t)$ as a function of its state $b(t)\dots b(t\text{-}Q)$. Such structure is known as a trellis coded modulation in communication theory [22], and the study of the performance of such systems is linked to the study of the embedded convolutional code.

In order to identify chaotic maps which optimize the performance of such schemes, the analysis of the system performance in terms of Minimum distance error events [22] can be carried out [21], illustrating close-to-BPSK performance for the Bernoulli shift map, while for the tent map, the performance is bad.

It turns out that performance can be improved beyond BPSK by combining the classical CSK approach shown in Figure 3 with the small perturbation control such that both inputs (the control sequence and the sequence switching the maps) is driven by the same (payload) data [21] and an appropriate choice of the maps f_0 and f_1. This principle is illustrated in Figure 4 below.

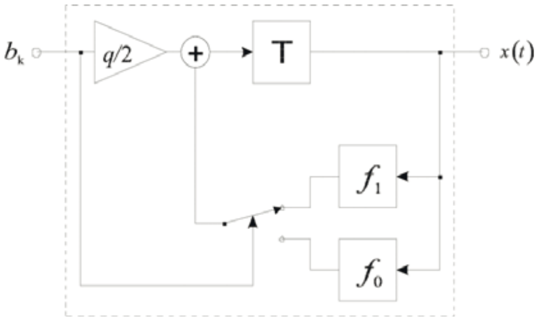

Fig. 4. Block diagram of a CSK transmitter with small perturbation controlled symbolic dynamics

This underlines again our answers to the questions 1 and 2 as given above, as for the given example they have both been addressed in a constructive way.

Conclusion

We have given a survey of approaches to communications with chaotic systems developed in the last 10 years. Given the generally mediocre performance of such approaches, we have identified two fundamental questions which should help us pinpoint the main problems with the existing approaches with the help of information theory and communication theory.

In particular it has been confirmed both in theory and by a constructive approach leading to a controlled variant of CSK, that indeed the information generated by free running chaotic systems (as captured by the Kolmogorov-Sinai entropy) is the problem, and the solution is a suitable control that makes this information part of the payload.

We believe that the approach demonstrated for the simple small perturbation controlled CSK transmitter is much more general and will find its application to "more physical" chaotic systems in the near future, leveraging the advantages of chaotic systems as given in the introduction.

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