

NOISE ANALYSIS AND OPTIMIZATION OF GENERAL OTA-C FILTERS

Slawomir Koziel,

*Faculty of Electronics, Telecommunications and Informatics, Gdansk University of Technology, Gdansk, Poland,
koziel@ue.eti.pg.gda.pl*

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ABSTRACT

In the paper, a general approach to noise analysis in continuous-time OTA-C filters is presented. Recently an increasing interest in the design of continuous-time filters based on the transconductancecapacitor (OTA-C) technique has been observed. The operational transconductance amplifiers (OTAs) offer a higher bandwidth than their voltage-mode counterparts, can be easily tuned electronically, and have a better suitability for operating in reduced supply environment. Due to this, high frequency integrated filters are mostly realized as the OTA-C ones. Based on a matrix description of a general OTA-C filter topology, a universal formulas for evaluating the noise in any OTA-C filter are derived. The presented formulas can be easily implemented and used in computer-aided analysis/optimization software. The accuracy of the proposed method is confirmed by comparison with SPICE simulation. The example of application for finding the minimum-noise 5th order multiple-loop feedback filters implementing Butterworth and Bessel transfer functions is given.

KEYWORDS: *OTA-C filters, noise analysis, filter optimization.*

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I. Introduction

Recently an increasing interest in the design of continuous-time filters based on the transconductance-capacitor (OTA-C) technique has been observed [1]-[3]. The operational transconductance amplifiers (OTAs) offer a higher bandwidth than their voltage-mode counterparts, can be easily tuned electronically, and have a better suitability for operating in reduced supply environment [4],[5]. Due to this, high frequency integrated filters are mostly realized as the OTA-C ones [6]. Although these filters offer excellent high frequency performance, their other properties in terms of low supply voltage, low power consumption, low sensitivity, low noise and large dynamic range, etc., still need improvements [5].

In this paper we deal with noise in OTA-C, which limits dynamic range of filters from below. It is important for filter design purposes to develop efficient tools for performing noise analysis of OTA-C filters. There have been several attempts to solve this problem described in the literature [7]-[10]. In this paper we propose a general approach to noise analysis in OTA-C filters based on the matrix description of OTA-C filters developed in [11]. The derived formulas can be applied to any know OTA-C filter architecture. They can also be easily implemented and used in computer-aided analysis/optimization software.

II. General topology of OTA-C filter

Figure 1 shows a general structure of a voltage-mode OTA-C filter. The filter contains n internal nodes denoted as x_i , $i=1,\dots,n$, n input transconductors G_{mbi} , a set of internal feedback and feedforward transconductors G_{mij} , an output summer consisting of transconductors G_{mci} and $-G_{mo}$ as well as a feedforward transconductor G_{md} . All transconductors form active network, while input capacitors C_{bi} , $i=1,\dots,n$ and capacitors C_{ij} , $1\leq i\leq j\leq n$ form *passive network*. It is easily seen that any OTA-C filter is a particular case of the general structure in Figure 1. A general filter structure in Fig.1 can be described by the following matrix equations [11]:

$$sT_C X = GX + B^T u_i \quad u_o = CX + Du_i \quad (1)$$

where u_i , u_o denote the input and output voltages, respectively, X is a vector of internal node voltages, and

$$\begin{aligned} T_C &= [T_{ij}]_{i,j=1}^n, \quad T_{ii} = C_{bi} + \sum_{j=1}^n C_{ij}, \quad i=1,\dots,n, \\ T_{ij} &= T_{ji} = -C_{ij}, \quad i,j=1,\dots,n, \quad i\neq j \\ G &= [G_{mij}]_{i,j=1}^n, \quad X = [x_1 \quad \dots \quad x_n]^T, \quad D = d = G_{md}/G_{mo} \\ C &= [c_1 \quad \dots \quad c_n], \quad c_i = G_{mci}/G_{mo}, \quad i=1,\dots,n \\ B &= [G_{mb1} + sC_{b1} \quad \dots \quad G_{mbn} + sC_{bn}] \end{aligned} \quad (2)$$

On the basis of (1) we can calculate the filter transfer function:

$$H(s) = \frac{u_o(s)}{u_i(s)} = C(sT_C - G)^{-1} B^T + D \quad (3)$$

Now, let us denote adjoint matrix of $sT_C - G$ as \tilde{A} where

$$\tilde{A}(s) = \text{adj}(sT_C - G) = [\tilde{A}_{ij}(s)]_{i,j=1}^n \quad (4)$$

This allows us to rewrite H in the form:

$$H(s) = [\det(sT_C - G)]^{-1} \sum_{i,j=1}^n c_i (G_{mbj} + sC_{bj}) \tilde{A}_{ij}(s) + d \quad (5)$$

Note that many filter structures have only one input transconductor (i.e. no input signal distribution), a trivial output summer (i.e. one of the internal nodes is the output of the filter), and no input capacitors. This means that $B = [0 \dots 0 \quad G_{mbk} \quad 0 \dots 0]$, $C = [0 \dots 0 \quad 1 \dots 0]$ - 1 at l -th position and $C_{bi}=0$ for $i=0,1,\dots,n$. Then, (5) reduces to the form of:

$$H(s) = \frac{G_{mbk} \tilde{A}_{lk}(s)}{\det(sT_C - G)} \quad (6)$$

Similar expression can be written for more general cases [11]. On the basis of the above expressions one can easily calculate the transmittance of any structure of OTA-C filter.

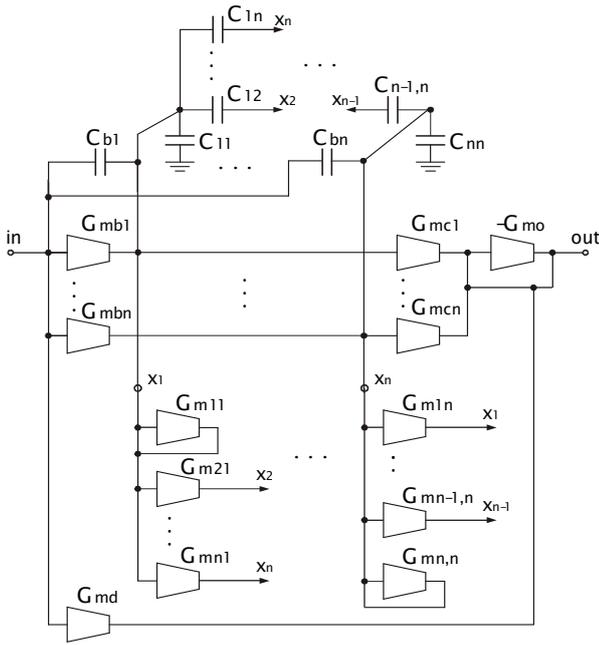


Fig. 1. General structure of voltage-mode OTA-C filter

III. Noise analysis of OTA-C filter

The output noise of any G_m -C filter is a combination of the noise contributions of its all transconductors. The noise in CMOS amplifier with transconductance g_m can be described in terms of an equivalent input referred noise voltage source v_n as shown in Figure 2.

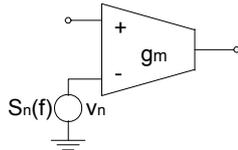


Fig. 2. Input noise source representation of noise in OTA

Spectral density $S_n(f)$ of the noise source can be modeled as [7]:

$$S_n(f) = S_t/g_m + S_f/f \quad (8)$$

where both S_t (thermal noise component) and S_f (flicker noise component) depend on amplifier topology. We shall assume that noise sources associated to different OTAs are statistically independent.

Our immediate goal is to obtain the explicit formula for output (and/or input) noise spectrum of the general OTA-C filter in Fig.1. In order to do this, one has to consider what is the contribution of the noise of each individual transconductance amplifier to the output noise spectrum of the filter. This can be modeled as shown in Fig.3. Let G_{mx} denote one of the filter transconductors (e.g. G_{mbi} , G_{mij} , etc.), which is connected to one of the nodes, say x_i (if the filter contains non-trivial output summer then it has additional output node which will be denoted as x_0). Denote by v_x the input referred noise voltage of the noise source corresponding to G_{mx} , whose spectral density is $S_{vx}(f)$. Transconductor G_{mx} injects its noise current $i_x = v_x G_{mx}$ into the node x_i . Spectral density $S_{ix}(f)$ of this current is given by

$$S_{ix}(f) = G_{mx}^2 S_{vx}(f) \quad (9)$$

The corresponding output noise voltage v_{ox} can be calculated as [7],[8]:

$$v_{ox} = i_x H_i = G_{mx} H_i v_x \quad (10)$$

where H_i is the current-to-voltage transfer function from node x_i to the output of the filter. Corresponding spectral density $S_{vox}(f)$ is given by the formula:

$$S_{vox}(f) = S_{ix}(f) |H_i(j2\pi f)|^2 = G_{mx}^2 S_{vx}(f) |H_i(j2\pi f)|^2 \quad (11)$$

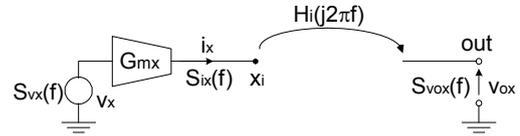


Fig. 3. Noise contribution of individual filter transconductor to the total output noise of the filter

It can be easily shown using the matrix equation (1) that the transfer functions $H_i(s)$, $i=1,2,\dots,n$ are components of the $1 \times n$ vector \mathbf{H}_{cv} defined as follows

$$\mathbf{H}(s) = \mathbf{C}(s\mathbf{T} - \mathbf{G})^{-1} \quad (12)$$

If non-trivial output summer is present (cf. Fig.1) then we need also the current-to-voltage transfer function from output node to itself, which is

$$H_0 = G_{mo}^{-1} \quad (13)$$

Thus, each filter transconductor injects its noise current into one of the internal nodes of the filter (or directly into the output node if the filter possesses nontrivial output summer). Subsequently, this current is converted into output noise voltage according to (11). In order to calculate the total output noise voltage of the filter we add noise spectra due to all transconductors. In general, the outputs of one input transconductor G_{mbi} , and n transconductors G_{mij} , $j=1, \dots, n$ are connected to each internal node x_i . In the presence of non-trivial output summer we have an additional node x_0 with outputs of transconductors G_{mcj} , $j=1, \dots, n$, G_{md} , and G_{mo} connected to it. Let us define auxiliary matrices

$$\begin{aligned} \mathbf{S}_t &= [S_{t,ij}]_{i,j=1}^n, \quad \mathbf{S}_f = [S_{f,ij}]_{i,j=1}^n, \\ \mathbf{S}_{tb} &= [S_{tb,1} \quad \dots \quad S_{tb,n}]^T, \quad \mathbf{S}_{fb} = [S_{fb,1} \quad \dots \quad S_{fb,n}]^T, \\ \mathbf{S}_{tc} &= [S_{tc,1} \quad \dots \quad S_{tc,n}], \quad \mathbf{S}_{fc} = [S_{fc,1} \quad \dots \quad S_{fc,n}], \\ \mathbf{S}_{td} &= S_{td}, \quad \mathbf{S}_{fd} = S_{fd}, \quad \mathbf{S}_{to} = S_{to}, \quad \mathbf{S}_{fo} = S_{fo} \end{aligned} \quad (14)$$

representing the thermal noise (subscript t) and $1/f$ noise (subscript f) of transconductors G_{mij} , G_{mbi} , G_{mci} , G_{md} and G_{mo} , respectively. Let us introduce the following notation:

$$\begin{aligned} \mathbf{G} &= [G_{mij}]_{i,j=1}^n, \quad \mathbf{B} = [G_{mb1} \quad \dots \quad G_{mbn}]^T, \\ \overline{\mathbf{C}} &= [G_{mc1} \quad \dots \quad G_{mcn}], \quad \overline{\mathbf{D}} = G_{md}, \quad \overline{\mathbf{O}} = G_{mo} \end{aligned} \quad (15)$$

Denote by \circ the Hadamard product of two matrices, i.e. for $\mathbf{P} = [p_{ij}]_{i,j=1}^n$ and $\mathbf{Q} = [q_{ij}]_{i,j=1}^n$ we have $\mathbf{P} \circ \mathbf{Q} = [p_{ij}q_{ij}]_{i,j=1}^n$ (the same definition holds, with obvious changes for $n \times 1$, $1 \times n$ and 1×1 matrices). Let $\hat{\mathbf{I}} = [1 \quad \dots \quad 1]^T$ be $n \times 1$ vector. Define function $F(\mathbf{P}, \mathbf{Q}, \mathbf{R})(x)$, where \mathbf{P} , \mathbf{Q} , and \mathbf{R} are matrices of the same dimension and x is a real variable:

$$F(\mathbf{P}, \mathbf{Q}, \mathbf{R})(x) = \mathbf{P} \circ (\mathbf{Q} + (2\pi/x)\mathbf{P} \circ \mathbf{R}) \quad (16)$$

It follows from (8) and (9) that spectral densities $S_i(\omega)$ of the total noise current injected into the nodes x_i , $i=1, \dots, n$ can be expressed, using (16), as components of the current spectral density vector \mathbf{S} , given by the formula

$$\begin{aligned} \mathbf{S}(\omega) &= [S_1(\omega) \quad \dots \quad S_n(\omega)]^T = \\ &= F(\overline{\mathbf{G}}, \mathbf{S}_t, \mathbf{S}_f)(\omega) \cdot \hat{\mathbf{I}} + F(\overline{\mathbf{B}}, \mathbf{S}_{tb}, \mathbf{S}_{fb})(\omega) \end{aligned} \quad (17)$$

Spectral density $S_0(\omega)$ of the noise current injected into the node x_0 (if output summer is present) is given by

$$\begin{aligned} S_0(\omega) &= F(\overline{\mathbf{C}}, \mathbf{S}_{tc}, \mathbf{S}_{fc})(\omega) \cdot \hat{\mathbf{I}} + \\ &+ F(\overline{\mathbf{D}}, \mathbf{S}_{td}, \mathbf{S}_{fd})(\omega) + F(\overline{\mathbf{O}}, \mathbf{S}_{to}, \mathbf{S}_{fo})(\omega) \end{aligned} \quad (18)$$

The spectrum density $S_{no}(\omega)$ of the total output noise voltage u_{no} can be then calculated as

$$S_{no}(\omega) = |H_{cv}(\omega)|^2 \mathbf{S}(\omega) + H_0^2 S_0(\omega) \quad (19)$$

where $|H_{cv}(\omega)|^2 = H_{cv}(j\omega) \circ H_{cv}(-j\omega)$, with H_{cv} and H_0 given by (12) and (13), respectively. In general, $S_o(\omega)$ is a rational function of ω with numerator and denominator of order not larger than $2n-1$ and $2n+1$, respectively. Formula (19) allows us to calculate the output noise spectrum of any OTA-C filter. In order to get the output noise voltage one needs to integrate (19) over the suitable frequency range. The equivalent input noise spectrum $S_{ni}(\omega)$ can be obtained by dividing (19) by the square of the transfer function of the filter given by (3). It is worth noting that because of matrix formulation, the presented formulas are particularly convenient to be implemented in a computer program which will allow us to carry out the noise analysis of arbitrary OTA-C filter.

In a special case when all transconductors in the filter are the same, each of the matrices \mathbf{S} in (14) is proportional to the unit matrix (i.e. with all entries equal to 1) $\tilde{\mathbf{I}}$ of the suitable dimension, e.g. we have $\mathbf{S}_t = S_t \tilde{\mathbf{I}}$, $\mathbf{S}_f = S_f \tilde{\mathbf{I}}$, where S_t , S_f are noise parameter of transconductor and $\tilde{\mathbf{I}}$ is $n \times n$ unit matrix. Then, equations (17) and (18) take the form

$$\mathbf{S}(\omega) = (\overline{\mathbf{G}}\hat{\mathbf{I}} + \overline{\mathbf{B}})S_t + \frac{2\pi}{\omega} (\overline{\mathbf{G}} \circ \overline{\mathbf{G}}\hat{\mathbf{I}} + \overline{\mathbf{B}} \circ \overline{\mathbf{B}})S_f \quad (20)$$

$$S_0(\omega) = \frac{\overline{\mathbf{C}}}{\hat{\mathbf{I}} + \overline{\mathbf{D}} + \overline{\mathbf{O}}} S_t + \frac{2\pi}{\omega} (\overline{\mathbf{C}} \circ \overline{\mathbf{C}}\hat{\mathbf{I}} + \overline{\mathbf{D}} \circ \overline{\mathbf{D}} + \overline{\mathbf{O}} \circ \overline{\mathbf{O}}) S_f \quad (21)$$

IV. Verification and application example

In this section we discuss an application example of the OTA-C filter noise analysis presented in Section III. Due to its generality and matrix formulation, the presented approach can be used to solve many optimization tasks such as proper choice of filter topology with respect to optimal noise performance or direct optimization of filter noise performance in various settings.

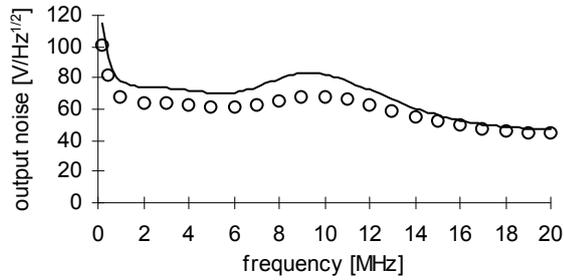
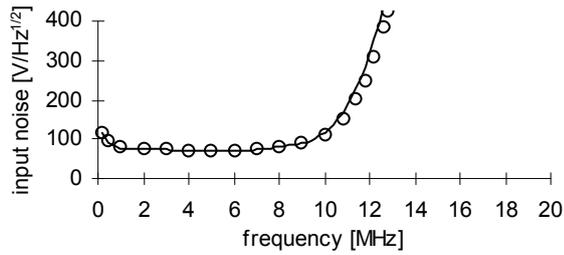


Fig. 8. Input and output noise spectrum vs. frequency for 8th order Butterworth filter; theory (line) and simulation (points)

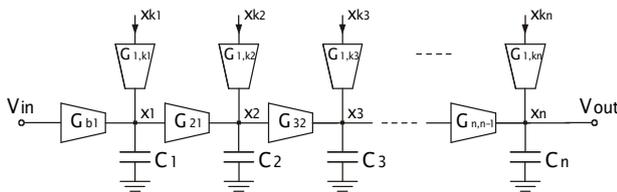


Fig. 9. A general structure of an n -th order all-pole canonical low-pass OTA-C filter

Table 1. Integrated input noise comparison for 5th order all-pole low-pass Butterworth and Bessel filters; best and worst topologies denoted by * and #, respectively

Filter structure	Normalized noise [dB]		Filter structure	Normalized noise [dB]	
	Butterworth	Bessel		Butterworth	Bessel
$F_{5 2,3,4,5,5}$	0.0	0.0	$F_{5 4,3,4,5,5}$	2.6	1.1
$F_{5 2,3,5,5,5}$	-0.2*	-0.4	$F_{5 4,3,5,5,5}$	3.2	1.3
$F_{5 2,4,4,5,5}$	2.4	0.4	$F_{5 4,4,4,5,5}$	5.7	2.4
$F_{5 2,4,5,5,5}$	2.9	-0.1	$F_{5 4,4,5,5,5}$	7.1	3.0
$F_{5 2,5,4,5,5}$	2.4	-0.5	$F_{5 4,5,4,5,5}$	5.3	1.3
$F_{5 2,5,5,5,5}$	3.0	-1.2*	$F_{5 4,5,5,5,5}$	6.8#	2.1
$F_{5 3,3,4,5,5}$	3.4	1.8	$F_{5 5,3,4,5,5}$	3.6	1.9
$F_{5 3,3,5,5,5}$	3.1	1.2	$F_{5 5,3,5,5,5}$	2.9	1.0
$F_{5 3,4,4,5,5}$	6.0	3.1#	$F_{5 5,4,4,5,5}$	6.2	2.9
$F_{5 3,4,5,5,5}$	6.7	2.8	$F_{5 5,4,5,5,5}$	6.7	2.5
$F_{5 3,5,4,5,5}$	5.5	2.1	$F_{5 5,5,4,5,5}$	5.9	1.9
$F_{5 3,5,5,5,5}$	6.2	1.7	$F_{5 5,5,5,5,5}$	6.3	1.3

V. Conclusions

In the paper, an efficient procedure for evaluating noise in general OTA-C filter has been proposed. It has been verified by comparing with SPICE simulation proving its accuracy. The derived formulas can be applied to any know OTA-C filter architecture and can be easily implemented and used in computer-aided analysis/optimization software.

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