NONIDEAL BEHAVIOUR OF OA-BASED ASTABLE MULTIVIBRATORS WITH APPLICATIONS IN LINEAR CAPACITANCE- AND INDUCTANCE-TO-TIME CONVERSIONS

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ABSTRACT

The nonideal behaviour of astable multivibra-tors based on Negative Impedance Converter (NIC) type configuration is approached taking into consideration several nonidealities among which the slew-rate of the voltage operational amplifier (OA). Linear conversion of inductances, capacitances and resis-tances into time period is useful to measure impedances or physical quantities by means of impedance-type trans-ducers. Following a series of investigations on this topic, a thorough study of four configurations for linear inductance-time and capacitance-time conversions based on the astable multivibrator viewed as a NIC-type nonlinear oscillator. Four circuits were studied, two of them having grounded reactive elements. The results can be used for linear capacitance-to-time (C|T) and inductance-to-time (L|T) conversions.

KEYWORDS: *Negative Impedance Converter, operational amplifier, capacitance-to-time, inductance-to-time.*

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I. Introduction

Linear conversion of inductances, capacitances and resistances into time period is useful to measure impedances or physical quantities by means of impedance-type transducers. Following a series of investigations on this topic [2–4], a thorough study of four configurations for linear inductance-time and capacitance-time conversions based on the astable multivibrator viewed as a NIC-type nonlinear oscillator has been reported in [5]. The L|T converter proposed in [2] consisted of a single OA astable multivibrator having the capacitor simulated by a two OA gyrator loaded by the inductance to be measured. The much simpler technique presented in [3] consisted of a single OA astable multivibrator, three resistances and the (ungrounded) inductance to be measured. To eliminate the disadvantage of the ungrounded inductance, a two OA-s L|T converter was proposed in [4]. In [5], the general theory of multivibrators viewed as NIC-type configurations, and the formulae for the time period taking into consideration the losses of the reactive elements were presented. The theory was based on the fact that the simplest astable multivibrators consist of a parallel connection of an inductor (capacitor) and an N-type (S-type) resistance. The possibilities to obtain N- and S-type negative resistances using one OA-based NIC configuration by taking into consideration the OA output saturation resistances (which strongly influence the shape of the voltage-current characteristics $(u\ i)$ and thus the period of oscillation) have been studied. In this way, a systematic study of a single OA family of L|T and C|T converters has been performed, resulting in a new L|T converter which was not only very simple, but exhibited also the advantage of a grounded inductance. Later, in [6], it has been shown for one of the L|T converters derived in [5], that for small time periods, when the slew rates strongly affect the circuit behavior, the L|T conversion is linear as well.

The aim of this paper is to present several improved results regarding all four types of single OA-based astable multivibrators using NIC-type configurations, when running in those regimes where the OA slew rates strongly affect their behavior. Using a time-domain analysis, it will be shown that the C|T and L|T conversions are linear, irrespective of the fact that the reactive element is grounded or not.

II. Basic Circuit

All results refer to the general configuration shown in Fig.1 where the passive elements including the parasitic resistances (R_p) are specified as indicated in Tab.1. When the oscillation conditions are satisfied and the operating frequency is low, the OA output behaves roughly like a piece-wise constant voltage source, switching between positive and negative polarity.
However, when the operating frequency is sufficiently high, the OA output behaves as a voltage source v_S generating a triangular waveform (due to slew rate limitations) in series with an output resistance R_{OUT} .

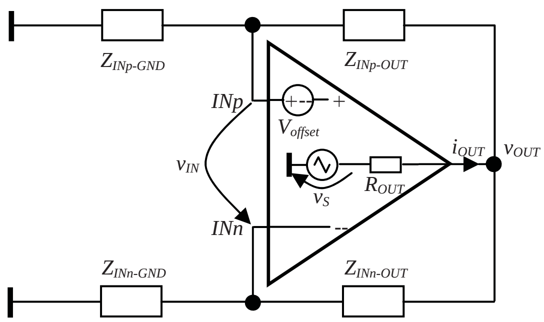

Fig.1 General schematic of a NIC-type astable multivibrator

Tab.1	C-type astable multivibrators		L-type astable multivibrators	
	C grounded	C floating	L grounded	L floating
$Z_{INp-GND}$	R_1	R_0	Series(L_0,R_p)	R_2
$Z_{INp-OUT}$	R_2	Parallel(C_0,R_p)	R_0	R_1
$Z_{INn-GND}$	Parallel(C_0,R_p)	R_2	R_1	R_0
$Z_{INn-OUT}$	R_0	R_1	R_2	Series(L_0,R_p)

In the following, the astable multivibrators based on the configuration depicted in Fig.1 will be studied under the hypothesis that the oscillation period is sufficiently low so that the OA output voltage v_{OUT} (current i_{OUT}) is below the saturation (limitation) values and v_S source exhibits a triangular waveform with slopes equal to the slew rates. Denoting by $V_{\min}$ and $V_{\max}$ the minimum and maximum values of the triangularly shaped voltage v_S , and by SR_p and SR_n its positive and negative slopes determined by the slew rates, then the rise and fall times are respectively $t_r = (V_{\max} - V_{\min})/SR_p$ and $t_f = (V_{\min} - V_{\max})/SR_n$, corresponding to an oscillation period of $T=t_r+t_f$. Using over the elementary oscillation period the following waveform for the voltage source

$$v_S(t)=v_{S(r)}(t)\left(\sigma(t)-\sigma(t-t_r)\right)+v_{S(f)}(t-t_r)\left(\sigma(t-t_r)-\sigma(t-t_r-t_f)\right)$$

where $v_{S(r)}(t)=V_{\min}+SR_p t$ and $v_{S(f)}(t)=V_{\max}+SR_n t$ are the rising and falling components, respectively, and $\sigma(t)$ denotes the Heaviside function, the OA input voltage, $v_{IN}(t)$, can be easily found in closed form, since the external network is linear. Once the input differential voltage $v_{IN}(t)$ is found, two sets of conditions can be used to determine the oscillation period:

- Crossing zero value conditions (**ZC**):

$$\begin{cases} v_{IN}(0)-V_{offset}=0 \\ v_{IN}(t_r)-V_{offset}=0 \end{cases} \text{ or equivalently } \begin{cases} v_{IN}(t_r)-V_{offset}=0 \\ v_{IN}(t_r+t_f)-V_{offset}=0 \end{cases}$$

- Piecewise constant polarity conditions (**PC**):

$$\begin{cases} v_{IN}(t)-V_{offset}>0, t\in(0,t_r) \\ v_{IN}(t)-V_{offset}<0, t\in(t_r,t_r+t_f) \end{cases}.$$

In the conditions above, V_{offset} denotes the OA input offset voltage, cf. Figure 1.

III. Input differential voltage and oscillation period

The linear network transfer functions from the output to the differential input of the OA, $H_{IN}(s)=V_{IN}(s)/V_S(s)$, has the form $H_{IN}(s)=k(s-\alpha)/(s+\beta)$, where $\alpha=1/(R_\alpha C_0)$, $\beta=1/(R_\beta C_0)$ or $\alpha=R_\alpha/L_0$, $\beta=R_\beta/L_0$, according to the type of reactive element used, while k and R_α, R_β are given in Tab.2. Applying Tab.1, we observe that the transfer function $H_{IN}(s)$ has the same form for any C- and L-type astable multivibrator, irrespective of the fact that the reactive element is grounded or not.

Tab.2	C-type astable multivibrator (grounded or floating)	L-type astable multivibrator (grounded or floating)
k	$\frac{1}{\left(1+\frac{R_2}{R_1}\right)\left(1+\frac{R_{OUT}}{R_0}\right)+\frac{R_{OUT}}{R_1}}$	$\frac{1}{1+\frac{R_1+R_{OUT}}{R_2}}$
R_α	$\frac{1}{\frac{R_2}{R_0R_1}-\frac{1}{R_p}}$	$\frac{R_0R_1}{R_2}-R_p$
R_β	$\frac{1}{\frac{1}{R_p}+\frac{1}{R_0+\frac{1}{\frac{1}{R_{OUT}}+\frac{1}{R_1+R_2}}}}$	$R_p+R_0+\frac{1}{\frac{1}{R_{OUT}}+\frac{1}{R_1+R_2}}$

The waveform of the OA input differential voltage for an oscillation period has the analytical expression:

$$v_{IN}(t)=v_{IN(r)}(t)\left(\sigma(t)-\sigma(t-t_r)\right)+v_{IN(f)}(t-t_r)\left(\sigma(t-t_r)-\sigma(t-t_r-t_f)\right)$$

where the two components corresponding to the voltage source rise and fall time intervals are respectively:

$$v_{IN(r)}(t)=k\left(\frac{SR_p}{\beta}\left(1+\frac{\alpha}{\beta}\right)-V_{\min}\frac{\alpha}{\beta}\right)-k\frac{\alpha}{\beta}SR_p t-$$
$$k\left(1+\frac{\alpha}{\beta}\right)\frac{SR_p-SR_n}{\beta}\frac{e^{\beta t_r}\left(e^{\beta t_f}-1\right)}{\left(e^{\beta(t_r+t_f)}-1\right)}e^{-\beta t}$$

$$v_{IN(f)}(t)=k\left(\frac{SR_n}{\beta}\left(1+\frac{\alpha}{\beta}\right)-V_{\max}\frac{\alpha}{\beta}\right)-k\frac{\alpha}{\beta}SR_n t-$$
$$k\left(1+\frac{\alpha}{\beta}\right)\frac{SR_n-SR_p}{\beta}\frac{e^{\beta t_f}\left(e^{\beta t_r}-1\right)}{\left(e^{\beta(t_f+t_r)}-1\right)}e^{-\beta t}$$

The (**ZC**) and (**PC**) conditions can be expressed as:

$$\begin{cases} v_{IN(r)}(0)-V_{offset}=0 \\ v_{IN(f)}(0)-V_{offset}=0 \end{cases} \text{ and } \begin{cases} v_{IN(r)}(t)-V_{offset}>0, t\in(0,t_r) \\ v_{IN(f)}(t)-V_{offset}<0, t\in(0,t_f) \end{cases}.$$

When (**ZC**) conditions are used the following, two equations in $V_{\min}$ and $V_{\max}$ are obtained:

$$V_{\min}=-V_{offset}\frac{\beta}{k\alpha}+$$
$$\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)\frac{SR_p\left(\coth\left(\frac{1}{2}\frac{V_{\min}-V_{\max}}{SR_n/\beta}\right)-1\right)+SR_n\left(\coth\left(\frac{1}{2}\frac{V_{\max}-V_{\min}}{SR_p/\beta}\right)+1\right)}{\coth\left(\frac{1}{2}\frac{V_{\min}-V_{\max}}{SR_n/\beta}\right)+\coth\left(\frac{1}{2}\frac{V_{\max}-V_{\min}}{SR_p/\beta}\right)},$$

$$V_{\max}=-V_{offset}\frac{\beta}{k\alpha}+$$
$$\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)\frac{SR_n\left(\coth\left(\frac{1}{2}\frac{V_{\max}-V_{\min}}{SR_p/\beta}\right)-1\right)+SR_p\left(\coth\left(\frac{1}{2}\frac{V_{\min}-V_{\max}}{SR_n/\beta}\right)+1\right)}{\coth\left(\frac{1}{2}\frac{V_{\max}-V_{\min}}{SR_p/\beta}\right)+\coth\left(\frac{1}{2}\frac{V_{\min}-V_{\max}}{SR_n/\beta}\right)},$$

which lead to the equation below for the voltage source peak-to-peak value ($V_{\max}-V_{\min}$) of the voltage source v_S:

$$\frac{1}{2}\frac{V_{\max}-V_{\min}}{SR_p-SR_n}\left(\coth\left(\frac{1}{2}\frac{V_{\max}-V_{\min}}{SR_p/\beta}\right)-\coth\left(\frac{1}{2}\frac{V_{\max}-V_{\min}}{SR_n/\beta}\right)\right)=\frac{1}{\alpha}+\frac{1}{\beta}.$$

Since the oscillation period is related to the voltage source peak-to-peak value through the relationship $T=(V_{\max}-V_{\min})/(1/SR_p-1/SR_n)$, the next equation can be derived for the oscillation period, T:

$$\frac{\beta T}{2K_{SR}}\left(\coth\left(\frac{\beta T}{4}\left(1-\sqrt{1-\frac{4}{K_{SR}}}\right)\right)+\coth\left(\frac{\beta T}{4}\left(1+\sqrt{1-\frac{4}{K_{SR}}}\right)\right)\right)=1+\frac{\beta}{\alpha},$$

where $K_{SR} = \quad -SR_p/SR_n \quad -SR_n/SR_p$ may be interpreted as a measure of the slew rate relative error, since $K_{SR} \geq 4$ (the equality is valid if $SR_p = -SR_n$).

Capacitor based astable multivibrators

For $\alpha = 1/(R_\alpha C_0)$ and $\beta = 1/(R_\beta C_0)$, the oscillation period is specified by $T = R_x C_0$, where R_x is the positive solution of the transcendental equation:

$$\frac{R_x}{2K_{SR}R_\beta}\left(\coth\left(\frac{R_x}{4R_\beta}\left(1-\sqrt{1-\frac{4}{K_{SR}}}\right)\right)+\coth\left(\frac{R_x}{4R_\beta}\left(1+\sqrt{1-\frac{4}{K_{SR}}}\right)\right)\right)=1+\frac{R_\alpha}{R_\beta}$$

The parameters $V_{\min}$ and $V_{\max}$ become respectively:

$$V_{\min} = -V_{offset}\frac{R_\alpha}{kR_\beta} +$$

$$\frac{1}{2}\frac{R_x C_0}{K_{SR}}\left(SR_p\left(\coth\left(\frac{R_x/(2R_\beta)}{1-SR_n/SR_p}\right)-1\right)+SR_n\left(\coth\left(\frac{R_x/(2R_\beta)}{1-SR_p/SR_n}\right)+1\right)\right)$$

$$V_{\max} = -V_{offset}\frac{R_\alpha}{kR_\beta} +$$

$$\frac{1}{2}\frac{R_x C_0}{K_{SR}}\left(SR_n\left(\coth\left(\frac{R_x/(2R_\beta)}{1-SR_p/SR_n}\right)-1\right)+SR_p\left(\coth\left(\frac{R_x/(2R_\beta)}{1-SR_n/SR_p}\right)+1\right)\right)$$

Inductor based astable multivibrators

For $\alpha = R_\alpha/L_0$ and $\beta = R_\beta/L_0$, the oscillation period is determined by $T = L_0/R_x$, where R_x is the positive solution

$$\frac{R_\beta}{2K_{SR}R_x}\left(\coth\left(\frac{R_\beta}{4R_x}\left(1-\sqrt{1-\frac{4}{K_{SR}}}\right)\right)+\coth\left(\frac{R_\beta}{4R_x}\left(1+\sqrt{1-\frac{4}{K_{SR}}}\right)\right)\right)=1+\frac{R_\beta}{R_\alpha}$$

The parameters $V_{\min}$ and $V_{\max}$ are then given by:

$$V_{\min} = -V_{offset}\frac{R_\beta}{kR_\alpha} +$$

$$\frac{1}{2}\frac{L_0}{K_{SR}R_x}\left(SR_p\left(\coth\left(\frac{R_\beta/(2R_x)}{1-SR_n/SR_p}\right)-1\right)+SR_n\left(\coth\left(\frac{R_\beta/(2R_x)}{1-SR_p/SR_n}\right)+1\right)\right)$$

$$V_{\max} = -V_{offset}\frac{R_\beta}{kR_\alpha} +$$

$$\frac{1}{2}\frac{L_0}{K_{SR}R_x}\left(SR_n\left(\coth\left(\frac{R_\beta/(2R_x)}{1-SR_p/SR_n}\right)-1\right)+SR_p\left(\coth\left(\frac{R_\beta/(2R_x)}{1-SR_n/SR_p}\right)+1\right)\right)$$

Discussion

1) From the above relations it can be seen that for each type of astable multivibrator, the oscillation period and the minimum and maximum values of v_S all *depend linearly* on the capacitance or inductance.
2) A necessary and sufficient condition for the existence of a (unique) positive solution of the transcendental equations is $R_\alpha > 0$ (since $R_\beta > 0$ according to Tab.2).
   If this condition is satisfied, it is easy to prove that the (**PC**) conditions are satisfied as well. The above constraint leads to corresponding constraints regarding the loss resistances of the reactive elements:
   - For C-type astable multivibrators: $R_p > R_1 R_0/R_2$
   - For L-type astable multivibrators: $R_p < R_1 R_0/R_2$.
3) The approximation $K_{SR} \simeq 4$ can be adopted, since a $\pm 10\%$ deviation of the slew rates around their nominal value leads to an increase of K_{SR} remaining within 1% from its minimal value $K_{SR} = 4$ (usually the slew rates errors are below the specified margins).
   In the case $K_{SR} = 4$ (i.e. $SR_p = -SR_n = SR$) gets verified, the above equations become much simpler:
   - For C-type astable multivibrators, the value of the resistance R_x is the solution of the equation $R_x \coth(R_x/(4R_\beta))/(4R_\beta) = 1 + R_\alpha/R_\beta$, while $T = R_x C_0$ and $V_{\min,\max} = \mp R_x C_0 SR/4 - V_{offset} R_\alpha/kR_\beta$. In the last expression, $V_{\min,\max}$ stands for $V_{\min}$ or $V_{\max}$, the leading negative or positive sign applying for $V_{\min}$ or $V_{\max}$, respectively.
   - For L-type astable multivibrators, the value of the resistance R_x is the solution of the equation $R_\beta \coth(R_\beta/(4R_x))/(4R_x) = 1 + R_\beta/R_\alpha$, while $T = L_0/R_x$ and $V_{\min,\max} = \mp L_0 SR/(4R_x) - V_{offset} R_\beta/kR_\alpha$.

   It is easily verified that the period depends neither on the slew rate values, nor on the offset voltage, but that it depends only on the OA output resistance and on the passive network elements. Moreover, the minimum and maximum values of v_S depend linearly on the slew rate and input offset voltage values.
4) It can be shown that R_x is bounded and can be approximated as follows:
   - In the case of C-based astable multivibrators,

$$K_{SR}R_\alpha < R_x < K_{SR}(R_\alpha + R_\beta) \text{ and } R_x \approx R_{xa} = \begin{cases} 2K_{SR}\sqrt{R_\alpha R_\beta}, R_\alpha \leq R_\beta \\ K_{SR}(R_\alpha + R_\alpha), R_\alpha \geq R_\beta \end{cases}.$$

The maximum of the relative error absolute value $|R_{xa}/R_x - 1|$ is $\sqrt{K_{SR}/3} - 1 \simeq \sqrt{4/3} - 1 < 16\%$ for $R_\alpha \ll R_\beta$, and decreases fast when R_α / R_β increases, cf. Fig.2-C(b).

- In the case of L-based astable multivibrators,

$$\frac{1/K_{SR}}{1/R_\alpha + 1/R_\beta} < R_x < \frac{R_\alpha}{K_{SR}} \text{ and } R_x \approx R_{xa} = \begin{cases} \frac{\sqrt{R_\alpha R_\beta}}{2K_{SR}}, R_\beta \le R_\alpha \\ \frac{1/K_{SR}}{1/R_\alpha + 1/R_\beta}, R_\beta \ge R_\alpha \end{cases}.$$

The maximum of the relative error absolute value $|R_{xa}/R_x - 1|$ is $1-\sqrt{3/K_{SR}} \simeq 1-\sqrt{3/4} < 14\%$ for $R_\beta \ll R_\alpha$, and decreases fast when R_β / R_α increases, cf. Fig.2-L(b).

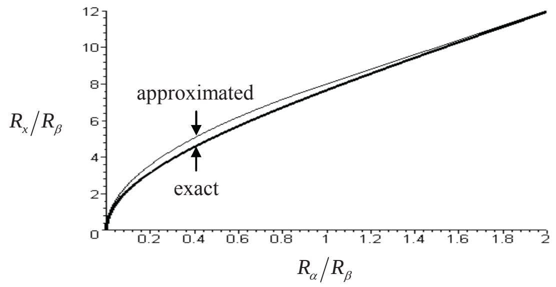

Fig.2-C(a) C-type astable multivibrators – dependence of the approximated (i.e. R_{xa}/R_β) and exact R_x/R_β ratios versus R_α/R_β

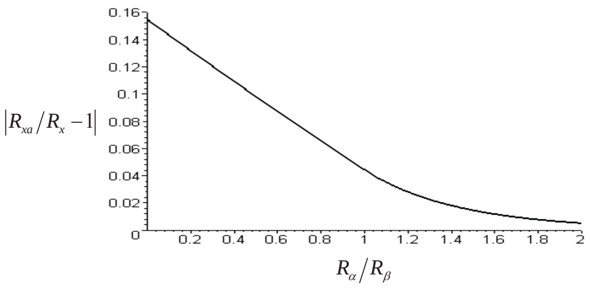

Fig.2-C(b) C-type astable multivibrators – dependence of the relative error absolute value $|R_{xa}/R_x - 1|$ on R_α/R_β

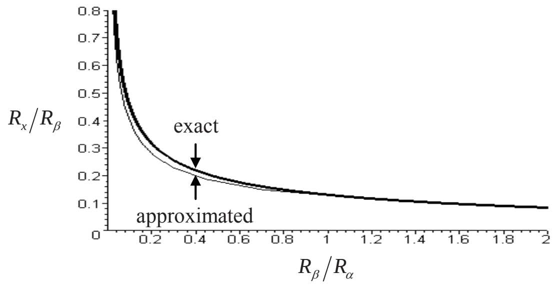

Fig.2-L(a) L-type astable multivibrators - dependence of the approximated (i.e. R_{xa}/R_β) and exact R_x/R_β ratios versus R_β/R_α

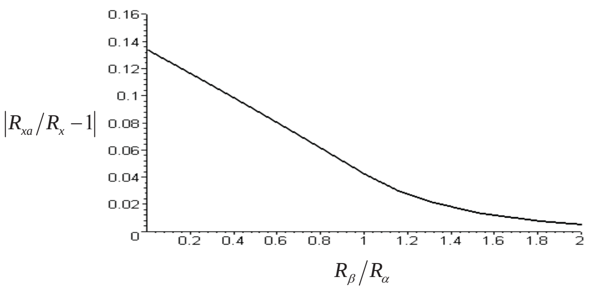

Fig.2-L(b) L-type astable multivibrators - dependence of the relative error absolute value $|R_{xa}/R_x - 1|$ on R_β/R_α

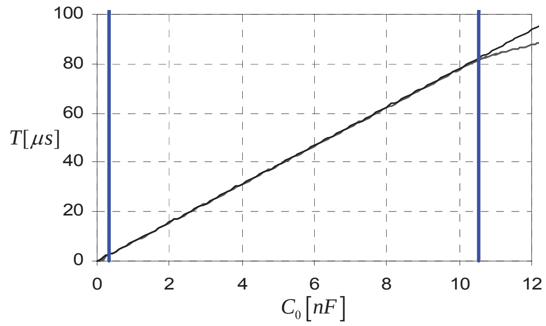

Fig.3-C(a) C-type astable multivibrators – dependence of the oscillation period on the capacitance

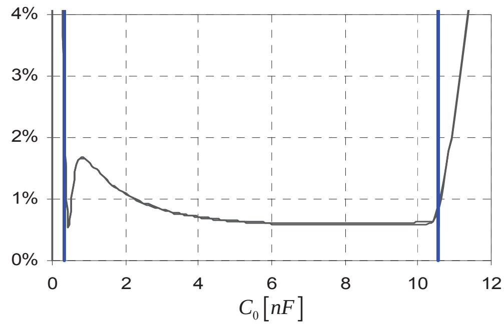

Fig.3-C(b) C-type astable multivibrators – dependence of the absolute value of the oscillation period relative error on the capacitance

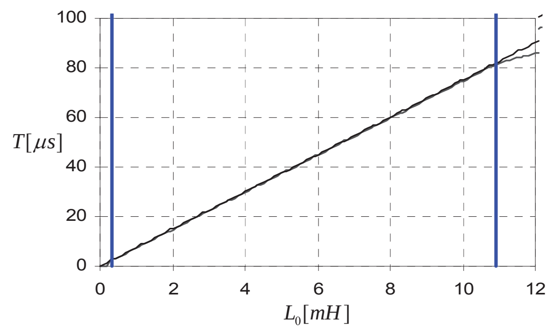

Fig.3-L(a) L-type astable multivibrators – dependence of the oscillation period on the inductance

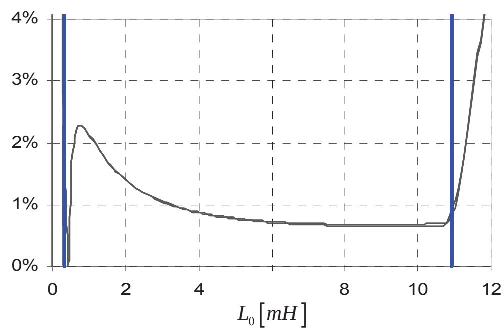

Fig.3-L(b) L-type astable multivibrators – dependence of the absolute value of the oscillation period relative error on the inductance

IV. Simulation results

A large number of parametric transient analyses have been performed using the PSPICE simulator. For the active device (OA), the PSPICE behavioral model of the general purpose LM741 amplifier has been used. Various supply voltages and R_0, R_1, R_2 values have been considered to better emphasize the behavior of the circuits. The simulation results match very well the theoretical model. As seen from Fig.3-C(a) and Fig.3-L(a), where the OA-external resistances have identical values ($1K\Omega$) and the OA is symmetrically biased ($\pm 10V$), the simulation results for the cases of grounded and floating reactive elements practically coincide and fit remarkably well to the theoretical behavior represented by the straight lines. This is confirmed by Fig.3-C(b) and Fig.3-L(b) depicting each the relative errors for both grounded and floating reactive element cases, which remain below 2.5% over the entire domain bounded by thick vertical lines within which the OA macro-model used is valid. The relative error abruptly increases for very small capacitance or inductance values for which the circuit approaches the stability limit, as well as for higher values where the OA output stage is working nonlinearly due to voltage saturation or current limitation.

V. Conclusions

Several results regarding OA-based C- and L-type astable multivibrators have been derived considering finite slew-rates and further nonidealities, i.e. OA output resistance and offset voltage, and parasitic resistances of the external reactive elements. The results show that linear C|T and L|T conversions are ensured even under such conditions. The scaling factor between the oscillation period and the value of the reactive element used depends only on the OA-external resistances and on *ratios of slew rates*, irrespective of the fact that the reactive element was grounded or not.

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