COMPLEX HYBRID SYMBOLIC ANALYSIS OF NONLINEAR ANALOG CIRCUITS DRIVEN BY MULTI-TONE SIGNALS

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ABSTRACT

The paper presents a general symbolic method for generating the complex hybrid matrix necessary for computing the periodic or nonperiodic steady-state response of a nonlinear analog circuit driven by multitone signals. This method is remarkable by its great efficiency and generality, and it is very useful in frequency-domain approach based on harmonic balance and least square approximation. For the general case of the nonperiodic steadystate response there are three basic methods: frequency-domain approach based on Voltera series; time-domain approach and a frequency-domain approach based on harmonic balance and least square approximation. The last one is significantly more efficient when the total number of nonlinear resistors, inductors and capacitors is significantly less than the total number of linear inductors and capacitors in the circuit, as is often the case in practice.

KEYWORDS: *Symbolic analysis, hybrid analysis, nonlinear analog circuits, multi-tone signals.*

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I. Introduction

For the general case of the nonperiodic steady-state response there are three basic methods: (1) frequency-domain approach based on Voltera series [1,10-12]; (2) time-domain approach [2,11-12], and (3) a frequency-domain approach based on harmonic balance and least square approximation [3]. The last one is significantly more efficient when the total number of nonlinear resistors, inductors, and capacitors is significantly less than the total number of linear inductors and capacitors in the circuit, as is often the case in practice.

This method requires the following steps: first – all nonlinear circuit elements and independent sources will be "extracted" from the circuit to the terminals, so that the resulting linear *n*-port *N* will contain only time-invariant circuit elements (resistors, inductors, capacitors, linear-controlled sources etc.); next – we must develop a frequency-domain analysis (ac analysis) of the substituted linear and time-invariant subcircuit (obtained by corresponding substitution of the nonlinear circuit elements with voltage or current sources) and an analysis based on harmonic method coupled with the least square approximation of the nonlinear subcircuit.

The frequency-domain (ac analysis) of the linear substituted *n*-port *N* can be efficiently implemented by computing the symbolic hybrid matrix corresponding to this subcircuit for each frequency $\hat{\omega}_k$. The method presented in this paper to generate the complex hybrid matrix for each frequency, necessary for calculating the periodic or not periodic steady-state response of a nonlinear circuit of great complexity, driven by multi-tone signals, is a very useful tool.

II. Frequency–domain analysis of nonlinear analog circuits driven by multi-tone signals

Let us consider the general case when the circuit contains nonlinear elements – resistors, inductors, capacitors, independent voltage and current sources having a dc component and *m* multi-tone frequencies $\omega_1, \omega_2, ..., \omega_m$, and linear circuit elements.

Let us extract all nonlinear elements and all independent sources to the ports. The nonlinear elements on the left ports are described as follows: voltage-controlled resistors: $\boldsymbol{i}_G = \hat{\boldsymbol{i}}_G(\boldsymbol{u}_G)$, voltage-controlled capacitors: $\boldsymbol{q}_C = \hat{\boldsymbol{q}}_C(\boldsymbol{u}_C)$, flux-controlled inductors: $\boldsymbol{i}_\Gamma = \hat{\boldsymbol{i}}_\Gamma(\boldsymbol{\varphi}_\Gamma)$, while the nonlinear elements on the right ports are described by the equations: current-controlled resistors: $\boldsymbol{u}_R = \hat{\boldsymbol{u}}_R(\boldsymbol{i}_R)$, charge-controlled capacitors: $\boldsymbol{u}_S = \hat{\boldsymbol{u}}_S(\boldsymbol{q}_S)$, current-controlled inductors: $\boldsymbol{\varphi}_L = \hat{\boldsymbol{\varphi}}_L(\boldsymbol{i}_L)$.

The linear *n*-port *N* may contain any time-invariant circuit element as: resistors, inductors, capacitors, linear-controlled sources etc. Substituting all the nonlinear elements from the left side by ideal voltage sources and all the nonlinear elements from the right side by ideal current sources, we obtain a linear and time-invariant circuit.

The vector of the unknowns has the following form:

$$\boldsymbol{x}(t) = \left[\boldsymbol{u}_G^{\mathrm{t}}(t), \boldsymbol{u}_C^{\mathrm{t}}(t), \boldsymbol{\varphi}_\Gamma^{\mathrm{t}}(t), \boldsymbol{i}_R^{\mathrm{t}}(t), \boldsymbol{q}_S^{\mathrm{t}}(t), \boldsymbol{i}_L^{\mathrm{t}}(t)\right]. \qquad (1)$$

It may be expressed as:

$$\boldsymbol{x}(t) = \boldsymbol{x}_0 + \sum_{k=1}^{M}\left(\boldsymbol{x}_{2k-1}\cos\hat{\omega}_k t + \boldsymbol{x}_{2k}\sin\hat{\omega}_k t\right) = \\ = \boldsymbol{x}_0 + \mathrm{Re}\left\{\sum_{k=1}^{M}\left(\boldsymbol{x}_{2k-1} - j\boldsymbol{x}_{2k}\right)e^{j\hat{\omega}_k t}\right\}. \qquad (2)$$

Expression (2) can be interpreted as a generalized finite Fourier series, where the frequency components include not only harmonics, but also intermodulation frequencies

$$\hat{\omega}_k \overset{\mathrm{d}}{=} m_{1k}\omega_1 + m_{2k}\omega_2 + ... + m_{pk}\omega_p, \qquad (3)$$

whith m_{ik}, $i = 1,2,\ldots,m$, satisfying the constraint

$$|m_{1k}| + |m_{2k}| + .. + |m_{mk}| \le p,$$

where p is the highest order of the frequency components considered (the components beyond this order are negligible).

For a given p, M represents the set of all frequency components satisfying the inequality constraint (4).

The steady-state response of the linear and time-invariant subcircuit, computed by frequency-domain techniques (ac analysis) with respect to the unknown Fourier coefficients $\boldsymbol{x}_0, \boldsymbol{x}_{2k-1}, \boldsymbol{x}_{2k}$; $k = 1,2,...,M$, is:

$$\begin{aligned} \boldsymbol{y}(t) &= \boldsymbol{y}_0 + \sum_{k=1}^{M} (\boldsymbol{y}_{2k-1}\cos\hat{\omega}_k t + \boldsymbol{y}_{2k}\sin\hat{\omega}_k t) = \\ &= \boldsymbol{y}_0 + \mathrm{Re}\left\{ \sum_{k=1}^{M} (\boldsymbol{y}_{2k-1} - j\boldsymbol{y}_{2k}) e^{j\hat{\omega}_k t} \right\}. \end{aligned} \quad (5)$$

The steady-state response of the nonlinear subcircuit in terms of the same unknown, can be expressed as:

$$\boldsymbol{y}'(t) = \boldsymbol{b}_0(\boldsymbol{x}) + \sum_{k=1}^{\hat{M}} (\boldsymbol{b}_{2k-1}(\boldsymbol{x}) \cdot \cos\hat{\omega}_k t + \boldsymbol{b}_{2k}(\boldsymbol{x}) \cdot \sin\hat{\omega}_k t) \quad (6)$$

In Eq. (6) $\boldsymbol{b}_0(\boldsymbol{x}), \boldsymbol{b}_{2k-1}(\boldsymbol{x}), \boldsymbol{b}_{2k}(\boldsymbol{x})$, $k = 1,2,...,\hat{M}$ are generalized Fourier coefficients of $\boldsymbol{y}'(t)$, which depend on the $\boldsymbol{x}_0, \boldsymbol{x}_{2k-1}, \boldsymbol{x}_{2k}$, $k = 1,2,...,M$, and $\hat{M} \geq M$ includes in steady-state response of nonlinear subcircuit all new frequency components generated by the nonlinearities.

In order to compute the coefficient vector $\boldsymbol{b}(\boldsymbol{x})$ we may apply the Discrete Fourier Transformation (DFT), if $\boldsymbol{y}'(t)$ is a periodic response or the least square method if it is not periodic [3].

Having the two responses $\boldsymbol{y}(t)$ and $\boldsymbol{y}'(t)$ expressed in terms of the unknowns $\boldsymbol{x}(t)$ we do the last step of the algorithm. From the substitution theorem and the method of harmonic balance it results that the coefficient $\boldsymbol{c}_k$ of each frequency component $\hat{\omega}_k$ of $\boldsymbol{y}(t) - \boldsymbol{y}'(t)$ is equal to zero. Because each coefficient $\boldsymbol{c}_k$ is a function of the $2M+1$ unknown Fourier coefficients $\boldsymbol{x}_0, \boldsymbol{x}_{2k-1}, \boldsymbol{x}_{2k}$, $k = 1,2,...,M$, for each nonlinear element we will obtain $2M+1$ independent nonlinear algebraic equations:

$$c_0(\boldsymbol{x}) = 0,\ c_1(\boldsymbol{x}) = 0,\ ....,\ c_{2M}(\boldsymbol{x}) = 0. \quad (7)$$

That means a total number of $n(2M+1)$ equations, where n is the total number of nonlinear elements, that can be expressed into a compact form by a nonlinear algebraic equation:

$$\boldsymbol{Fx} + \boldsymbol{S} - \boldsymbol{b}(\boldsymbol{x}) = \boldsymbol{0}\ . \quad (8)$$

This equation can be solved using the Newton-Raphson algorithm obtaining the independent variables $\boldsymbol{x}_0, \boldsymbol{x}_1, ..., \boldsymbol{x}_{2M}$ from equation (2). The first two terms of the equation (8) are obtained by ac analysis of the linear substituted subcircuit. Our main contribution in this paper is to develop an efficient method to compute these terms; in other words to compute the hybrid matrix of the substituted linear n-port N.

III. Generation of the hybrid matrix of the linear subcircuit at frequency $\hat{\omega}_k$

For a given nonlinear circuit a special tree -called the *normal tree* (*NT*)- is chosen [4,7,9]. The normal tree elements are selected in this strict order: all ideal independent or/and controlled voltage sources; all nonlinear elements extracted on the left side – called nonlinear elements of type one (the associated variables will have the subscript 1); linear capacitors, resistors, and inductors. The corresponding cotree will contain: all ideal independent or/and controlled current sources; all nonlinear elements extracted on the right side – called nonlinear elements of type two (the associated variables will have the subscript 2); linear capacitors, resistors, and inductors. The *NT* will not contain independent or controlled current sources. Let T be a normal tree and L its corresponding cotree.

Remark 1. When the controlling variables of the controlled sources are associated to the linear resistors, capacitors, inductors or they are voltages of the nonlinear elements of type one respectively currents of the nonlinear elements of type two, these variables can be simple expressed at each frequency $\hat{\omega}_k$, starting from their constitutive equations, in respect of the independent variables $(\underline{\boldsymbol{U}}_{1\_k}, \underline{\boldsymbol{I}}_{2\_k})$ and of the input quantities $(\underline{\boldsymbol{E}}_k, \underline{\boldsymbol{J}}_k)$.

Applying the superposition theorem in complex it results at each frequency $\hat{\omega}_k$:

$$\begin{bmatrix} \underline{\boldsymbol{I}}_{1\_k} \\ \underline{\boldsymbol{U}}_{2\_k} \end{bmatrix} = \begin{bmatrix} \underline{\boldsymbol{Y}}_{1\_k,1\_k} & \underline{\boldsymbol{B}}_{1\_k,2\_k} \\ \underline{\boldsymbol{A}}_{2\_k,1\_k} & \underline{\boldsymbol{Z}}_{2\_k,2\_k} \end{bmatrix} \begin{bmatrix} \underline{\boldsymbol{U}}_{1\_k} \\ \underline{\boldsymbol{I}}_{2\_k} \end{bmatrix} + \begin{bmatrix} \underline{\boldsymbol{Y}}_{1\_k,e} & \underline{\boldsymbol{B}}_{1\_k,j} \\ \underline{\boldsymbol{A}}_{2\_k,e} & \underline{\boldsymbol{Z}}_{2\_k,j} \end{bmatrix} \begin{bmatrix} \underline{\boldsymbol{E}}_k \\ \underline{\boldsymbol{J}}_k \end{bmatrix}, \quad (9)$$

where, for example, $\underline{\boldsymbol{B}}_{1\_k,2\_k}$ $(\underline{\boldsymbol{A}}_{2\_k,1\_k})$ represents the matrix of the complex current (voltage) gains of the nonlinear elements of type one (two) in respect of the nonlinear elements of type two (one), and $\underline{\boldsymbol{U}}_{1\_k}$ $(\underline{\boldsymbol{I}}_{2\_k})$ is the complex voltage (current) vector of the nonlinear elements of type one (two). The meaning of the other variables results from their subscripts.

In equation (9) the state variables $\left(\underline{\boldsymbol{U}}_{1\_k}, \underline{\boldsymbol{I}}_{2\_k}\right)$ and the complementary state variables $\left(\underline{\boldsymbol{I}}_{1\_k}, \underline{\boldsymbol{U}}_{2\_k}\right)$ can be expressed as:

$$\underline{\boldsymbol{U}}_{1\_k} = \begin{bmatrix} \underline{\boldsymbol{U}}_{G\_k} \\ \underline{\boldsymbol{U}}_{C\_k} \\ j\hat{\omega}_k \underline{\boldsymbol{\Phi}}_{\Gamma\_k} \end{bmatrix} = \begin{bmatrix} \boldsymbol{u}_{G,2k-1} - j\boldsymbol{u}_{G,2k} \\ \boldsymbol{u}_{C,2k-1} - j\boldsymbol{u}_{C,2k} \\ j\hat{\omega}_k \left(\boldsymbol{\varphi}_{\Gamma,2k-1} - j\boldsymbol{\varphi}_{\Gamma,2k}\right) \end{bmatrix}, \quad (10)$$

$$\underline{\boldsymbol{I}}_{2\_k} = \begin{bmatrix} \underline{\boldsymbol{I}}_{R\_k} \\ j\hat{\omega}_k \underline{\boldsymbol{Q}}_{S\_k} \\ \underline{\boldsymbol{I}}_{L\_k} \end{bmatrix} = \begin{bmatrix} \boldsymbol{i}_{R,2k-1} - j\boldsymbol{i}_{R,2k} \\ j\hat{\omega}_k \ \boldsymbol{q}_{S,2k-1} - j\boldsymbol{q}_{S,2k} \\ \boldsymbol{i}_{L,2k-1} - j\boldsymbol{i}_{L,2k} \end{bmatrix}, \quad (11)$$

$$\underline{\boldsymbol{I}}_{1\_k} = \begin{bmatrix} \underline{\boldsymbol{I}}_{G\_k} \\ j\hat{\omega}_k \underline{\boldsymbol{Q}}_{C\_k} \\ \underline{\boldsymbol{I}}_{\Gamma\_k} \end{bmatrix} = \begin{bmatrix} \boldsymbol{i}_{G,2k-1} - j\boldsymbol{i}_{G,2k} \\ j\hat{\omega}_k \left(\boldsymbol{q}_{C,2k-1} - j\boldsymbol{q}_{C,2k}\right) \\ \boldsymbol{i}_{\Gamma,2k-1} - j\boldsymbol{i}_{\Gamma,2k} \end{bmatrix}, \quad (12)$$

$$\underline{\boldsymbol{U}}_{2\_k} = \begin{bmatrix} \underline{\boldsymbol{U}}_{R\_k} \\ \underline{\boldsymbol{U}}_{S\_k} \\ j\hat{\omega}_k \underline{\boldsymbol{\Phi}}_{L\_k} \end{bmatrix} = \begin{bmatrix} \boldsymbol{u}_{R,2k-1} - j\boldsymbol{u}_{R,2k} \\ \boldsymbol{u}_{S,2k-1} - j\boldsymbol{u}_{S,2k} \\ j\hat{\omega}_k \ \boldsymbol{\varphi}_{L,2k-1} - j\boldsymbol{\varphi}_{L,2k} \end{bmatrix}. \quad (13)$$

The characteristic equations of the linear circuit elements included both in tree and in cotree, for each frequency $\hat{\omega}_k$, are:

$\underline{\boldsymbol{U}}_{Z_t\_k} = \underline{\boldsymbol{Z}}_{t\_k} \underline{\boldsymbol{I}}_{Z_t\_k}$, respectively $\underline{\boldsymbol{I}}_{Z_t\_k} = \underline{\boldsymbol{Y}}_{t\_k} \underline{\boldsymbol{U}}_{Z_t\_k}$,

$\underline{\boldsymbol{U}}_{Z_c\_k} = \underline{\boldsymbol{Z}}_{c\_k} \underline{\boldsymbol{I}}_{Z_c\_k}$, respectively $\underline{\boldsymbol{I}}_{Z_c\_k} = \underline{\boldsymbol{Y}}_{c\_k} \underline{\boldsymbol{U}}_{Z_c\_k}$,

where

$$\underline{Z}_{t(c)\_k} = R + j\left(\hat{\omega}_k L - \frac{1}{\hat{\omega}_k C}\right),$$
$$\text{and } \underline{Y}_{t(c)\_k} = G + j\left(-\frac{1}{\hat{\omega}_k L} + \hat{\omega}_k C\right). \quad (14)$$

Denoting:

$$\underline{\boldsymbol{H}}_{11\_k} = \underline{\boldsymbol{Y}}_{1\_k,1\_k}, \ \underline{\boldsymbol{H}}_{12\_k} = \underline{\boldsymbol{B}}_{1\_k,2\_k},$$
$$\underline{\boldsymbol{H}}_{21\_k} = \underline{\boldsymbol{A}}_{2\_k,1\_k}, \ \underline{\boldsymbol{H}}_{22\_k} = \underline{\boldsymbol{Z}}_{2\_k,2\_k},$$

$$\underline{\boldsymbol{J}}_{s\_k} = \underline{\boldsymbol{Y}}_{1\_k,e} \underline{\boldsymbol{E}}_k + \underline{\boldsymbol{B}}_{1\_k,j} \underline{\boldsymbol{J}}_k, \underline{\boldsymbol{E}}_{s\_k} = = \underline{\boldsymbol{A}}_{2\_k,e} \underline{\boldsymbol{E}}_k + \underline{\boldsymbol{Z}}_{2\_k,j} \underline{\boldsymbol{J}}_k, \quad (15)$$

the equation (9) becomes:

$$\begin{bmatrix} \underline{\boldsymbol{I}}_{1\_k} \\ \underline{\boldsymbol{U}}_{2\_k} \end{bmatrix} = \begin{bmatrix} \underline{\boldsymbol{H}}_{11\_k} & \underline{\boldsymbol{H}}_{12\_k} \\ \underline{\boldsymbol{H}}_{21\_k} & \underline{\boldsymbol{H}}_{22\_k} \end{bmatrix} \begin{bmatrix} \underline{\boldsymbol{U}}_{1\_k} \\ \underline{\boldsymbol{I}}_{2\_k} \end{bmatrix} + \begin{bmatrix} \underline{\boldsymbol{J}}_{s\_k} \\ \underline{\boldsymbol{E}}_{s\_k} \end{bmatrix}. \quad (16)$$

In order to extract the frequency $\hat{\omega}_k$ from the unknown vector, we have to do the following notations:

$$\underline{\boldsymbol{X}}_{1\_k} = \begin{bmatrix} \boldsymbol{u}_{G,2k-1} - j\boldsymbol{u}_{G,2k} \\ \boldsymbol{u}_{C,2k-1} - j\boldsymbol{u}_{C,2k} \\ \boldsymbol{\varphi}_{\Gamma,2k-1} - j\boldsymbol{\varphi}_{\Gamma,2k} \end{bmatrix} = \boldsymbol{X}_{1\_k}^R + j\boldsymbol{X}_{1\_k}^I,$$

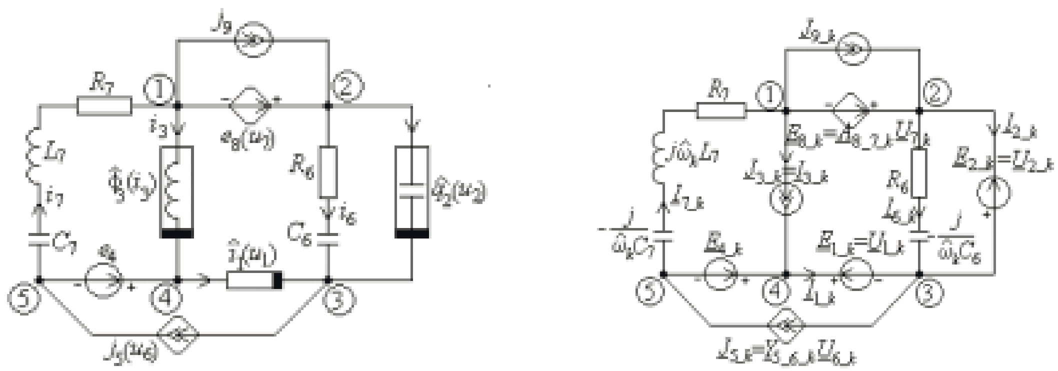
$$\underline{\boldsymbol{X}}_{2\_k} = \begin{bmatrix} \boldsymbol{i}_{R,2k-1} - j\boldsymbol{i}_{R,2k} \\ \boldsymbol{q}_{S,2k-1} - j\boldsymbol{q}_{S,2k} \\ \boldsymbol{i}_{L,2k-1} - j\boldsymbol{i}_{L,2k} \end{bmatrix} = \boldsymbol{X}_{2\_k}^R + j\boldsymbol{X}_{2\_k}^I. \quad (17)$$

Now the linear and time-invariant response of n-port N can be expressed by the explicit equation:

$$\begin{bmatrix} \boldsymbol{i}_1(t) \\ \boldsymbol{u}_2(t) \end{bmatrix} = \begin{bmatrix} \boldsymbol{i}_{1\_0} \\ \boldsymbol{u}_{2\_0} \end{bmatrix} + Re\left(\sum_{k=1}^{M} \begin{bmatrix} \underline{\boldsymbol{I}}_{1\_k} \\ \underline{\boldsymbol{U}}_{2\_k} \end{bmatrix} e^{j\hat{\omega}_k t}\right) =$$

$$= Re\left\{\begin{bmatrix} \boldsymbol{i}_{10} \\ \boldsymbol{u}_{20} \end{bmatrix} + \sum_{k=1}^{M}\left(\begin{bmatrix} \underline{\boldsymbol{F}}_{11k} & \underline{\boldsymbol{F}}_{12k} \\ \underline{\boldsymbol{F}}_{21k} & \underline{\boldsymbol{F}}_{22k} \end{bmatrix} \begin{bmatrix} \boldsymbol{X}_{1\_k}^R + j\boldsymbol{X}_{1\_k}^I \\ \boldsymbol{X}_{2\_k}^R + j\boldsymbol{X}_{2\_k}^I \end{bmatrix}\right) e^{j\hat{\omega}_k t}\right\} +$$

$$+ Re\left(\sum_{k=1} \begin{bmatrix} \underline{\boldsymbol{J}}_{s\_k} \\ \underline{\boldsymbol{E}}_{s\_k} \end{bmatrix} e^{j\hat{\omega}_k t}\right), \quad (18)$$



(a) (b)

Fig. 2. a) Initial circuit diagram; и) The substituted circuit diagram.

where: $\boldsymbol{i}_{1\_0}, \boldsymbol{u}_{2\_0}$ represent the dc unknowns which will be calculated from the dc analysis; $\boldsymbol{X}^R_{1\_k}, \boldsymbol{X}^I_{1\_k}, \boldsymbol{X}^R_{2\_k}$, and $\boldsymbol{X}^I_{2\_k}$ are the unknowns of the ac analysis corresponding to the frequency $\hat{\omega}_k$ and they are defined by (17); the submatrices $\underline{\boldsymbol{F}}_{ij\_k}$ are obtained from the submatrices $\underline{\boldsymbol{H}}_{ij\_k}$ multiplying by $j\hat{\omega}_k$ the components corresponding to the third row of $\underline{\boldsymbol{X}}_{1\_k}$ or the second row of $\underline{\boldsymbol{X}}_{2\_k}$. Equation (18) contains the first two terms of the equation (8).

We have implemented the above algorithm in a very efficient and flexible computing program that automatically generates the normal tree, the hybrid matrix and the source vector. Then it performs the frequency-domain analysis of the linear substituted *n*-port *N* for each frequency $\hat{\omega}_k$.

IV. Example

Let us consider the nonlinear circuit represented in Fig.1,a where: $i_1 = \hat{i}_1(u_1), q_2 = \hat{q}_2(u_2)$ and $\varphi_3 = \hat{\varphi}_3(i_3)$. Substituting the nonlinear circuit elements by ideal independent sources, according to the procedure described in Section II, we obtain the linear circuit in Fig. 1, b. The normal tree is made up of the branches: $\{\underline{E}_{4\_k}, \underline{E}_{8\_k}, \underline{E}_{1\_k}, \underline{E}_{2\_k}\}$, and the corresponding cotree contains the branches: $\{\underline{J}_{3\_k}, \underline{J}_{5\_k}, \underline{J}_{9\_k}, \underline{Z}_{6\_k}, \underline{Z}_{7\_k}\}$.

Applying the computing program, we obtain the complex hybrid matrix in the following form:

$$\underline{\boldsymbol{H}}_{11k} = \begin{bmatrix} \dfrac{1}{\underline{Z}_{7k}(\underline{A}_{87k}-1)} & \underline{Y}_{56k} - \dfrac{1}{\underline{Z}_{7k}(\underline{A}_{87k}-1)} \\ -\dfrac{1}{\underline{Z}_{7k}(\underline{A}_{87k}-1)} & -\dfrac{1}{\underline{Z}_{6k}} + \dfrac{1}{\underline{Z}_{7k}(\underline{A}_{87k}-1)} \end{bmatrix},$$

$$\underline{\boldsymbol{H}}_{12k} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad \underline{\boldsymbol{H}}_{21k} = \begin{bmatrix} \dfrac{1}{\underline{A}_{87k}-1} & -\dfrac{1}{\underline{A}_{87k}-1} \end{bmatrix}, \quad \underline{\boldsymbol{H}}_{22k} = [0].$$

The source vectors have the following expressions:

$$\underline{\boldsymbol{J}}_{sk} = \begin{bmatrix} -\dfrac{\underline{E}_{4k}}{\underline{Z}_{7k}(\underline{A}_{87k}-1)} \\ \dfrac{\underline{E}_{4\_k}}{\underline{Z}_{7k}(\underline{A}_{87k}-1)} \end{bmatrix}, \quad \underline{\boldsymbol{E}}_{sk} = \begin{bmatrix} -\dfrac{\underline{A}_{87k}\underline{E}_{4k}}{\underline{A}_{87k}-1} \end{bmatrix}.$$

V. Conclusion

The paper presents a simple method to generate the complex hybrid matrix and the complex source vectors, in symbolic form, for the nonlinear circuit driven by multi-tone signal analysis. This is very useful for steady-state response computation and it may be successfully integrated in the frequency-domain approach based on harmonic balance and least square method. The method is remarkable by its great efficiency and generality.

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