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SPECTRUM CHARACTERISTICS IN TELEVISION SYSTEMS OBJECTS WITH QUASI-PERIODIC PHASE-TIME CONVERSIONS OF VIDEO INFORMATION

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ABSTRACT

Quasiperiodic phase-time transformations of video information can be carried out both due to the objects corresponding displacements themselves, and through the necessary displacements organization, for example, in the signal sensor of television systems. The essence of this transformation type is relative, time-periodic displacements of controlled objects in raster space on the photosensitive surface of the signal sensor, accompanied by changes in the position of the object relative to the optical axis. In this case, large-scale changes in coordinates also affect the angle of each element of the object image. Consequently, the action of the operator of "periodic circular displacements" is associated with the occurrence of the effect of relative suppression of the upper spatial frequencies of the initial influence. The article shows that in terms of the effect on the spatio-temporal spectrum of the initial impact, it is advisable to classify the operator of "periodic linear displacements" as operators of the differential type, and the operator of "periodic circular displacements" as operators of the integral type. Any operator of integral type can be reduced to differential type and vice versa. An example is considered where the operator of periodic circular displacements is easily converted to a differential type when introducing a return circular displacement along the same trajectory.

KEYWORDS: *Television, Phase-Time Transformations, Video information.*

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1 Introduction

Quasiperiodic phase-time transformations of video information can be carried out both due to the objects corresponding displacements themselves, and through the organization of the necessary displacements, for example, in the signal television systems sensor [1]. The essence of this type of transformation is relative, time-periodic displacements of controlled objects in raster space on the photosensitive surface of the signal sensor, accompanied by changes in the position of the object relative to the optical axis. In this case, large-scale changes in coordinates also affect the angle of each element of the object image [2].

Any operator of integral type can be reduced to differential type and vice versa. The operator of periodic circular displacements discussed above, for example, is easily converted to a differential type by introducing a return circular displacement along the same trajectory.

2 Methods and solutions

Consider, for example, the impact displacement operator in an optical system.

$$X = x + \rho(t)\cos\varphi(t),\ \mathrm{Y} = \mathrm{y} + \rho(t)\sin\varphi(t);$$
$$x = X - \rho(t)\cos\varphi(t),\ \mathrm{y} = \mathrm{Y} - \rho(t)\sin\varphi(t),\ \mathrm{t} = \mathrm{t}. \quad (1)$$

Let's represent the initial impact *O*(*x,y,t*) in form [3]:

$$\mathrm{O(x,y,t)} = \mathrm{O_I}(x,y,t) + \mathrm{O_{II}(\text{-}x,y,t)} + \mathrm{O_{III}(x,\text{-}y,t)} + \mathrm{O_{IV}(\text{-}x,\text{-}y,t)}$$
$$\{0 \le \mathrm{x} < \infty; 0 \le \mathrm{y} < \infty; 0 \le \mathrm{t} < \infty\}.$$
$$\Im_c = \left| \frac{\partial(x,y,t)}{\partial(X,Y,t)} \right| = 1.$$

Let's find the Jacobian.

Next, let us analyze the spectral influence transformation operator in OS:

$$O(x,y,t) \rightarrow S(P_x, P_y, P_t);$$
$$O(X,Y,t) \rightarrow S_{\ddot{i}}(P_x, P_y, P_t).$$
$$S_{\ddot{i}}(P_x,P_y,P_t) = \iiint_{G_i} O_I(X,Y,t)\exp\left\{-P_t t - P_x\left[X - \rho(t)\cos\varphi(t)\right] - P_y\left[Y - \rho(t)\sin\varphi(t)\right]\right\} dXdYdt +$$
$$+ \iiint_{G_i} O_{II}(X,Y,t)\exp\left\{-P_t t - P_x\left[X - \rho(t)\cos\varphi(t)\right] - P_y\left[Y - \rho(t)\sin\varphi(t)\right]\right\} dXdYdt +$$
$$+ \iiint_{G_i} O_{III}(X,Y,t)\exp\left\{-P_t t - P_x\left[X - \rho(t)\cos\varphi(t)\right] - P_y\left[-Y - \rho(t)\sin\varphi(t)\right]\right\} dxdydt +$$
$$+ \iiint_{G_i} O_{IV}(X,Y,t)\exp\left\{-P_t t - P_x\left[X - \rho(t)\cos\varphi(t)\right] - P_y\left[Y - \rho(t)\sin\varphi(t)\right]\right\} dxdydt. \quad (2)$$

We will assume that impact is localized in the first quadrant of *X,Y* plane. Moreover, in accordance with (2), all components except the first are equal to 0.

Then

$$S_{\ddot{i}}(P_x,P_y,P_t) = \iiint_{G_i} O_I(X,Y,t)\exp\left\{-P_t t - P_x x - P_y y\right\}\exp\left\{P_x\rho(t)\cos\varphi(t) + P_y\rho(t)\sin\varphi(t)\right\} dxdydt. \quad (3)$$

The resulting expression (3) indicates that the resulting space-time spectrum is the Laplace transform [4] from product of functions O_I (x,y,t) and exp [$P_x\rho(t)\cos\varphi(t)$ $P_y(t)\sin\varphi(t)$

$$S_n(P_x,P_y,P_t)=L\{O_I(X,Y,t)\}\otimes L\{\exp[+P_x\rho(t)\cos\varphi(t)+P_y\rho(t)\sin\varphi(t)]\}. \qquad (4)$$

In accordance with (4), the structure S_P (P_x, P_y, P_t) is determined by the structure of initial impact spatio-temporal spectrum and function spectrum

$$\mathrm{f(t)=\exp\left[P_x\rho(t)\cos\varphi(t)+P_y\rho(t)\sin\varphi(t)\right]} \qquad (5)$$

Let us analyze S_P (P_x, P_y, P_t) under the condition $\omega_x\rho(t)\cos\varphi(t)$ <<1 and $\omega_x\rho$ (t)$\sin\varphi(t)$ <<1. Let us represent function $f(t)$ in the form of a Maclaurin series and, taking into account the above condition, we limit ourselves to only three terms of series:

$$f(t)=\exp\left[P_x\rho(t)\cos\varphi(t)+P_y\rho(t)\sin\varphi(t)\right]\approx\left[1-P_x\rho(t)\cos\varphi(t)+\frac{1}{2}P_x{}^2\rho^2(t)\cos^2\varphi(t)...\right]\cdot$$
$$\cdot\left[1-P_y\rho(t)\sin\varphi(t)+\frac{1}{2}P_y{}^2\rho^2(t)\sin^2\varphi(t)...\right]\approx\left[1-P_x\rho(t)\cos\varphi(t)+\frac{1}{2}P_x{}^2\rho^2(t)\cos^2\varphi(t)-\right.$$
$$\left.-P_y\rho(t)\sin\varphi(t)+P_xP_y\rho^2(t)\cos\varphi(t)\sin\varphi(t)+\frac{1}{2}P_y{}^2\rho^2(t)\sin^2\varphi(t)...\right] \qquad (6)$$

Taking into account (11) and (14) we obtain

$$S_{\ddot{I}}(P_x,P_y,P_t)=L\{O(X,Y,t)\}-L\left\{\frac{\partial\left[\rho(t)\cos\varphi(t)O_I(X,Y,t)\right]}{\partial x}\right\}+\frac{1}{2}L\left\{\frac{\partial\left[\rho^2(t)\cos^2\varphi(t)O_I(X,Y,t)\right]}{\partial^2 x}\right\}-$$
$$-L\left\{\frac{\partial\left[\rho(t)\sin\varphi(t)O_I(X,Y,t)\right]}{\partial y}\right\}+\frac{1}{2}L\left\{\frac{\partial\left[\rho^2(t)\sin\varphi(t)O_I(X,Y,t)\right]}{\partial^2 y}\right\}+L\left\{\frac{\partial\left[\rho^2(t)\cos\varphi(t)\sin\varphi(t)O_I(X,Y,t)\right]}{\partial x\partial y}\right\}.. \qquad (7)$$

The resulting expression (7) shows that during phase-temporal transformations of an impact with a spectrum localized in the region of relatively low spatial frequencies, the summation of the initial impact with time-varying difference components takes place, reflecting the corresponding spatio-temporal changes in the impact. The degree of change in the spectrum of the initial impact in this case does not depend on the coordinates of the object's points in space.

In accordance with the structure of relationship (7), cases of relatively insignificant spatial displacements of the impact in time can be modeled by summing the initial impact with its differential ones in space components modulated in time by functions of the form $\rho^*\cos^n\varphi(t)$, $\varphi^m\cos^p\varphi(t)$... reflecting the intensity and direction of the displacement.

In this case, accompanying changes in the temporal spectrum of the impact occur only in the region of upper spatial frequencies. For an initial impact that is constant over time, the result of the transformation is the appearance of time frequency components in the resulting spectrum. The time spectrum of the transformed impact is completely determined by the type of functions $\rho(t)$ and $\varphi(t)$.

Special cases of phase-time transformations are cases with $\varphi(t)$= const or $\rho(t)$ = const.

When analyzing the option with $\varphi(t)$ = const = φ, it is advisable to transition from coordinates *x, y* to coordinates

$$\overline{\mathrm{X}}=\mathrm{X}\cos\varphi+\mathrm{Y}\sin\varphi \text{ и } \overline{\mathrm{Y}}=\mathrm{X}\sin\varphi-\mathrm{Y}\cos\varphi.$$

According to (9)

$$\overline{X} = x\cos\varphi + y\sin\varphi\ ,\ \overline{Y} = x\sin\varphi - y\cos\varphi;$$
$$x = \overline{X}\cos\varphi + \overline{Y}\sin\varphi - \rho(t)\cos\varphi\ ,\ y = \overline{X}\sin\varphi - \overline{Y}\cos\varphi - \rho(t)\sin\varphi. \quad (8)$$

Let us find the Jacobian $\Im_c = -1$. Then

$$S_{\ddot{I}}(P_x, P_y, P_t) = -\iiint_{G_i^*} O_I(\bar{X}, \bar{Y}, t)\exp\Big\{-P_t t - \big[P_x\cos\varphi + P_y\sin\varphi\big]\cdot\bar{X} - \big[P_x\sin\varphi - P_y\cos\varphi\big]\cdot\bar{Y} +$$
$$+\rho(t)\big[P_y\sin\varphi + P_x\cos\varphi\big]\Big\}d\bar{X}d\bar{Y}dt.$$

$$P_{\bar{x}} = P_x\cos\varphi + P_y\sin\varphi, \quad P_{\bar{y}} = P_x\sin\varphi - P_y\cos\varphi. \quad (9)$$

Taking into account (9) we obtain

$$S_{\ddot{I}}(P_{\bar{x}}, P_{\bar{y}}, P_t) = \iiint_{G_i^*} O_I(\bar{X}, \bar{Y}, t)\exp\Big[-P_t t - P_{\bar{x}}\bar{X} - P_{\bar{y}}\bar{Y} + \rho(t)P_{\bar{x}}\Big]d\bar{X}d\bar{Y}dt. \quad (10)$$

According to (10), in the $\bar{X}, \bar{Y}, t$ space, phase-time transformations are manifested by changes in time along $\bar{X}$ and $\bar{Y}$ axes.

$$S_n(P_x, P_y, P_t) = -L\{O_I(X, Y, t)\} \otimes L\{\exp[\rho(t)P_x]\}. \quad (11)$$

The latter significantly simplifies the definition of the form $S_P(P_{\bar{X}}, P_{\bar{Y}}, P_t)$ for specific implementations of ρ(*t*). In the case of time-invariant impact O_I (*x, y, t*) = O_I (*x, y*) time spectrum of the transformed impact is completely determined by spectrum of function $f(t) = \exp[\rho(t)P_{\bar{X}}]$.

Let us consider the space-time spectrum features for the case of operator of “periodic linear displacements” implementation of the initial influence, to which expressions (8-11) correspond in the case

$$\rho(t) = \bar{Y}\cos\omega_0 t.$$

Considering this

$$R(P_x, P_y, P_t) = L\{\exp[\rho(t)P_y]\} = \frac{1}{P_y}\cdot\frac{1}{P_x}\cdot\left[\frac{1}{P_t} + 2\sum_{k=1}^{\infty}\frac{P_t\Im_k\left(k\frac{P_y Y_0\omega_0}{P_t}\right)}{P_t^2 + \omega_0^2 k^2}\right]. \quad (12)$$

It is well known $F(P) = \frac{P_t}{P_t^2 + \omega_0^2 k^2}$. is the image of $S(t)$ = cos$\omega_0 t$. Taking this into account, when moving to real variables $\omega, \omega_{\bar{x}}, \omega_{\bar{y}}$, expression (20) can be transformed to the form [3,5]:

$$R(\omega_{\bar{x}}, \omega_{\bar{y}}, \omega) = \left\{ \pi\delta(\omega) + \frac{1}{j\omega} + 2\sum_{k=1}^{\infty} \left[\pi\delta(\omega - k\omega_0) + \pi\delta(\omega + k\omega_0) \right] \Im_k \left(k \frac{\omega_{\bar{y}} Y_0 \omega_0}{\omega} \right) \right\} \cdot$$

$$\cdot \left[\pi\delta(\omega_{\bar{x}}) + \frac{1}{j\omega_x} \right] \cdot \left[\pi\delta(\omega_{\bar{y}}) + \frac{1}{j\omega_{\bar{y}}} \right]. \tag{13}$$

The resulting relationship (13) indicates the occurrence of maxima in the time spectrum at frequencies $\omega = \pm k\omega_0$, where k =1, 2, 3...

Let's present (13) as follows:

$$R(\omega_{\bar{x}}, \omega_{\bar{y}}, \omega) = R_0(\omega_{\bar{x}}, \omega_{\bar{y}}, \omega) + R_{\Gamma}(\omega_{\bar{x}}, \omega_{\bar{y}}, \omega) = \left[\pi\delta(\omega) + \frac{1}{j\omega} \right] \left[\pi\delta(\omega_{\bar{x}}) + \frac{1}{j\omega_{\bar{x}}} \right] \left[\pi\delta(\omega_{\bar{y}}) + \frac{1}{j\omega_{\bar{y}}} \right] +$$

$$+ \left\{ 2\sum_{k=1}^{\infty} \left[\pi\delta(\omega - k\omega_0) + \pi\delta(\omega + k\omega_0) \right] \Im_k \left(k \frac{\omega_{\bar{y}} \bar{Y}_0 \omega_0}{\omega} \right) \right\} \cdot \left[\pi\delta(\omega_{\bar{x}}) + \frac{1}{j\omega_{\bar{x}}} \right] \left[\pi\delta(\omega_{\bar{y}}) + \frac{1}{j\omega_{\bar{y}}} \right], \tag{14}$$

R_0 (ω_x, ω_y, ω) corresponds to the spectral density of a single jump. In the case of $R_P(\omega_{\bar{x}}, \omega_{\bar{y}}, \omega) \to 0$, the initial impact $O_t(\bar{X}, \bar{Y}, t)$ is multiplied by a single space-time jump. The spectrum of influence does not change.

Expression (10) taking into account (13, 14) can be represented by two components

$$S_{\Pi}(P_{\bar{x}}, P_{\bar{y}}, P_t) = -\left[S(\omega_{\bar{x}}, \omega_{\bar{y}}, \omega) + S(\omega_{\bar{x}}, \omega_{\bar{y}}, \omega) \otimes R_{\Pi}(\omega_{\bar{x}}, \omega_{\bar{y}}, \omega) \right]. \tag{15}$$

For odd K = (2n+1)=1,3,5... the function $\Im_k \left(k \frac{\omega_y Y_0 \omega_0}{\omega} \right) = -\Im_k \left(-k \frac{\omega_y Y_0 \omega_0}{\omega} \right)$ is odd.

The implementation of convolution in relation (15) is equivalent in this case to the differentiation of the original effect along the $\bar{Y}$ axis. For K = 2n = 2,4,6,8,... the function $\Im_k \left(k \frac{\omega_y Y_0 \omega_0}{\omega} \right) = -\Im_k \left(-k \frac{\omega_y Y_0 \omega_0}{\omega} \right)$ is even.

In this case, the implementation of convolution is associated with the appearance in the resulting spectrum S_P (ω_x, ω_y, ω) of components causing changes in time in the structure of the spatial spectrum in the region of low spatial frequencies.

According to expression (14), operator (8) does not lead to a change in the structure of the space-time spectrum of the initial impact at $\omega \to 0$ and $\omega_{\bar{y}} \to 0$. Under conditions of fixation ω = kω0, there is a discrete structure in the spatial spectrum $S_P(\omega_{\bar{x}}, \omega_{\bar{y}}, k\omega_0)$, the zeros of which are determined by the roots

$$\Im_k \left(k \frac{\omega_y Y_0 \omega_0}{\omega} \right)$$

the corresponding Bessel Function.

For ρ(t) = const = a

$$X = x + a\cos\varphi(t), Y = y + a\sin\varphi(t);$$
$$x = X - a\cos\varphi(t), y = Y - a\sin\varphi(t);$$
$$\Im_c = 1.$$

When analyzing this option, it is advisable to transition from *x,y* coordinates to coordinates

$$\bar{X} = X + a\cos\varphi(t) + a, \bar{Y} = y + a\sin\varphi(t) + a;$$
$$X = \bar{X} - a\cos\varphi(t) - a, y = \bar{Y} - a\sin\varphi(t) - a;$$
$$\Im_C = 1.$$

Then

$$S_{\ddot{\imath}}(P_x, P_y, P_t) = \iiint_{G_i^*} O_I(\bar{X}, \bar{Y}, t)\exp\left[P_x a\cos\varphi(t) + P_x a + P_y a\sin\varphi(t) + P_y a\right]$$
$$\exp\left[-P_x\bar{X} - P_y\bar{Y} - P_t t\right]d\bar{X}d\bar{Y}dt = \exp\left[P_x a + P_y a\right]\iiint_{G^*} O_I(\bar{X}, \bar{Y}, t)$$
$$\exp\left[P_x a\cos\varphi(t) + P_y a\sin\varphi(t)\right]\exp\left[-P_x\bar{X} - P_y\bar{Y} - P_t t\right]d\bar{X}d\bar{Y}dt. \quad (16)$$

According to (16)

$$S_n(P_x, P_y, P_t) = L\{O_I(X, Y, t)\} \otimes L\{\exp[P_x a\cos\varphi(t) + P_y a\sin\varphi(t)]\}. \quad (17)$$

Therefore, in the case of time-invariant exposure
O_I (x, y, t) = O_I (x, y) the time spectrum of transformed impact is completely determined by the function spectrum

$$\mathrm{f(t)} = \exp\left[\mathrm{P_x a}\cos\varphi(\mathrm{t}) + \mathrm{P_y a}\sin\varphi(\mathrm{t})\right].$$

Let us consider the features of the function $f(t)$ spectrum, for example, with $f(t) = \omega_0 t$. Taking into account the relations known from the Bessel functions theory [4,5], we use the relations known from theory of modified Bessel functions:

$$\exp(z\cos\theta) = I_0(z) + 2\sum_{k=1}^{\infty} I_k(z)\cos(k\theta);$$
$$\exp(z\sin\theta) = I_0(z) + 2\sum_{k=1}^{\infty}(-1)^k I_{2k+1}(z).$$
$$\sin\{(2k+1)\theta\} + 2\sum_{k=1}^{\infty}(-1)^k I_{2k}(z)\cos(2k\theta).$$

we get:

$$f(t)=\left[I_0(P_x a)+2\sum_{k=1}^{\infty} I_k(P_x a)\cos k\omega_0 t\right]$$

$$\left[\begin{array}{c} I_0(P_y a)+2\sum_{k=0}^{\infty}(-1)^k I_{2k+1}(P_y a)\sin(2k+1)\omega_0 t+ \\ +2\sum_{k=1}^{\infty}(-1)^k I_{2k}(P_y a)\cos 2\omega_0 t \end{array}\right]. \qquad (18)$$

Taking into account the connection between modified Bessel functions and ordinary Bessel functions, expression (18) can be transformed to the form:

$$f(\omega_x,\omega_y,t)=\left[\Im_0(\omega_x a)+2\sum_{k=1}^{\infty}(i)^{-k}(-1)^k \Im_k(\omega_x a)\cos k\omega_0 t\right]\left[\Im_0(\omega_y a)-\frac{2}{i}\sum_{k=0}^{\infty}(-1)^k\cdot\right.$$
$$\left.\cdot\Im_{2k+1}(\omega_y a)\sin\left\{(2k+1)\omega_0 t\right\}-2\sum_{k=1}^{\infty}(-1)^k \Im_{2k}(\omega_y a)\cos(2k\omega_0 t)\right]. \qquad (19)$$

3 Conclusion

The resulting final relation (19) corresponds to the operator of "periodic circular displacements" ($\rho(t)$ = a,φ (t) = $\omega_0 t$) and indicates the appearance in the resulting time spectrum of an infinite number of components with time frequencies that are multiples of the frequency ω_0. The considered operator of "periodic circular displacements" differs significantly from the operator of "periodic linear displacements" in structure of the space-time spectrum [6]. According to the structure of expression (19).

This operator, in particular, is characterized by a sharp relative increase in the amplitude of spectral components in the region of low spatial frequencies $\omega_x \to 0$, $\omega_y \to 0$. A comparison of the properties of the Bessel functions and the structure of the resulting relation (19) allows us, in addition, to conclude that for parameter values $\omega_x a$ and $\omega_y a \leq 0.45$, the influence of the circular displacement operator can be ignored.

influence on the spatial spectrum of the initial impact. With a further increase in the parameters $\omega_x a$ and $\omega_y a$, there is a sharp drop in the relative amplitude of the spectrum components, respectively, in the region of upper spatial frequencies.

Consequently, the operator of "periodic circular displacements" action is associated with the occurrence of relative suppression effect of the initial influence upper spatial frequencies. Based on the initial impact spatio-temporal spectrum effect, it is advisable to classify the operator of "periodic linear displacements" as differential type operators, and operator of "periodic circular displacements" as integral type operators.

It should be noted that any operator of integral type can be reduced to differential type and vice versa. The operator of periodic circular displacements discussed above, for example, is easily converted to a differential type by introducing a return circular displacement along the same trajectory.

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