

ACCOUNTING FOR THE ANTENNA LOSSES INFLUENCE ON INTERACTION CHANNELS IN THE RECEIVING ANTENNAS MULTICHANNEL THEORY

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ABSTRACT

The multichannel theory of receiving antennas is currently the most effective approach to analyzing the receiving and scattering properties of receiving antennas. According to this theory, the scattering field of the receiving antenna is considered as the sum of the information component of the scattering field and the residual component. These components form interaction channels through which power is taken from the plane wave by the receiving antenna. All the numerous results on the development of this theory that we obtained earlier related to the cases of ideally conducting receiving antennas. In such antennas, the only absorber of energy taken by the antenna from a plane wave is the antenna load. This work is devoted to the study of the influence of thermal losses in the receiving antenna on the connection of the and components of the receiving antenna with the wave incident on it. In this work, a two-channel model of a receiving antenna is constructed, which makes it possible to take into account the influence of thermal losses in the antenna walls on the energy of the process of interaction of the scattering field components of the receiving antenna with an incident plane wave. Based on this model, it is shown that the presence of thermal losses leads both to a change in the amplitude of the information component of the scattering field of the receiving antenna itself, which causes a change in the extinction power taken from a plane wave via the information channel, and to a change in the number of power consumers transmitted via the interaction information channel. A simple analytical expression has been obtained that allows, based on the known efficiency of the antenna, to carry out a numerical assessment of the influence of losses in the antenna on the power transmitted from a plane wave through the interaction information channel.

KEYWORDS: *Two-Channel Model, Scattering Diagram, Optical Theorem, Receiving Antenna, Information Component, Residual Component.*

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1 Introduction

The multichannel theory of receiving antennas [1-4] is based on the idea of dividing the antenna's scattering field into various components and connecting each of them with its own interaction channel, through which power is supplied from a plane wave incident on the antenna to the receiving antenna. Using a different number of components depending on the expansion basis, it became possible to study various aspects of the interaction of the receiving antenna load with different components of the stray field. It has been shown that the power delivered to the load is not related to the entire power dissipated by the antenna, but only to that part of it that can be described by the parameters of the antenna in transmit mode. This part of the scattering field was later called the information component of the scattering field of the receiving antenna, since the transfer of information from a plane wave to the antenna is associated with the transfer of power to the antenna load.

Thus, it was possible to show that the information and electrodynamic connection between a plane wave and an antenna are not the same thing. For example, if the receiving antenna is irradiated from the direction in which the zero of the radiation pattern is formed in the transmission mode, then the information component of the scattering field of the receiving antenna in this case is equal to zero and no power is supplied to the load, and, therefore, no information is transmitted from the plane wave to the antenna load.

In this case, the electrodynamic connection between the plane wave and the receiving antenna is preserved, since the plane wave induces currents on the surface of the antenna, which create a field scattered by the antenna and, therefore, the antenna selects power from the plane wave equal to the power that it scatters.

To explain why only the information component of the scattering field of the receiving antenna participates in the transfer of energy to the receiving antenna load, it was necessary to decompose the remaining part into three components, each of which individually forms its own interaction channel and contributes to the current flowing through the antenna load, and the sum of these currents in the load is zero [5]. In [6], a four-channel model of a receiving antenna was proposed, which describes the energy connections between a plane wave incident on the receiving antenna and the stray field components introduced in [5].

As can be seen from the above, by now the multichannel theory of receiving antennas has been quite well developed and has made it possible to obtain a number of new fundamental results. The need for further development of the multichannel theory of receiving antennas is due to the fact that all of the above results were obtained under the assumption that there are no losses in the receiving antennas under study and the only device that can absorb the power taken from a plane wave is the antenna load.

The presence of losses in the antenna walls leads to a change in the distribution of currents on the antenna walls, which in some cases can lead to a significant change in the main parameters of the antenna (antenna radiation pattern, directional coefficient, antenna efficiency). In addition, in a lossy antenna, part of the field penetrates into the walls of the antenna and is lost in them.

That is, in addition to the antenna load, a new energy consumer appears, which can significantly change the entire energy of interaction channels. Therefore, the multi-channel models that we mentioned above require additional consideration when trying to apply them to lossy antennas.

In this work, we investigated the influence of thermal losses in the walls of the receiving antenna on the energy of interaction of the components of the scattering field of the receiving antennas with the field of the incident plane wave. The multichannel theory of receiving antennas was used as a research method. When using the multichannel method, it is very important to successfully divide the total scattering field of the antenna into components, select their number and properties. The number of interaction channels depends on the number of components of the scattering field, since each component is associated with its own interaction channel, through which power is taken from a plane wave incident on the receiving antenna.

Increasing the number of channels makes it possible to find new connections between the parameters of receiving antennas and the fields scattered by it. At the same time, the model becomes more complex, which makes it difficult to obtain simple analytical results that could be used for further research. Therefore, from the very beginning, it was important to precisely formulate which connections between losses in the antenna walls and scattered fields would be of primary interest to us.

Since the transfer of information from a plane wave to the antenna load occurs only through the interaction information channel, we were naturally interested in the effect of losses in the receiving antenna on the operation of this interaction channel. Since the main parameter of the antenna, which characterizes the losses in it, is the efficiency of the antenna, the problem posed above can be formulated more precisely. Investigate the influence of the antenna efficiency on the operation of the information channel of interaction of the receiving antenna.

2 Methods and solutions

Let us consider the initial problem of determining influence of receiving antenna losses on the operation of interaction information channel in the following formulation. The receiving antenna, which has thermal losses, is located in free space with the parameters $\varepsilon_0, \mu_0, \sigma = 0$. A plane wave falls on it, the electric and magnetic vectors of which are described by the relations

$$\dot{\vec{E}}^i(\vec{n}_0, \vec{r}_0) = \dot{\vec{e}}_0 \exp[-ik(\vec{n}_0 \vec{r}_0)r], \quad (1)$$

$$\dot{\vec{H}}^i(\vec{n}_0, \vec{r}_0) = \frac{1}{Z_0} [\vec{n}_0, \dot{\vec{e}}_0] \dot{\vec{e}}_0 \exp[-ik(\vec{n}_0 \vec{r}_0)r] \quad (2)$$

where \vec{r}_0 – unit vector in a spherical coordinate system (r, θ, φ) , and \vec{n}_0 – unit vector of the incident wave propagation direction, $\dot{\vec{e}}_0$ – unit complex vector ($|\dot{\vec{e}}_0| = 1$), characterizing the primary wave polarization, and

$$\dot{\vec{e}}_0(\vec{r}_0) = \cos \eta \vec{\theta}_0 + \exp[i\beta] \sin \eta \vec{\varphi}_0, \quad (3)$$

where parameter $\eta (0 \leq \eta \leq \pi/2)$ determines the vector components amplitudes ratio $\vec{p}(\vec{r}_0)$; β – phase shift between components; $\vec{\theta}_0$ u $\vec{\varphi}_0$ – vectors of the spherical coordinate system, Z_0 – wave impedance of free space, and $k = \frac{2\pi}{\lambda}$ – wave number.

When a plane wave falls on the receiving antenna, a scattered field from the antenna appears $\dot{\vec{E}}_s(\vec{n}_0, \vec{r}_0)$, which in general can be written in the form

$$\dot{\vec{E}}_s(\vec{n}_0, \vec{r}_0) = \dot{\vec{A}}_s(\vec{n}_0, \vec{r}_0) \frac{\exp(-ikr)}{r}, \quad (4)$$

where $\dot{\vec{A}}_s(\vec{n}_0, \vec{r}_0)$ – complex antenna scattering diagram.

In what follows, it is assumed that we know all the parameters of the receiving antenna in transmission mode. That is, its complex vector radiation pattern is known $\vec{F}(-\vec{n}_0)$, antenna efficiency and load reflection coefficients \tilde{A} and antenna input γ .

When studying the energy of receiving antennas using the multichannel theory method, the total scattering field of the antenna is divided into components, the number and properties of which depend on the specific problem being solved. For example, in [6], the field scattered by the antenna was represented in the form

$$\begin{aligned} \vec{E}_s(\vec{n}_0, \vec{r}_0) &= \vec{E}_{\text{inf}}^{\text{bas}}(\vec{n}_0, \vec{r}_0) + \vec{E}_{\text{inf}}^{\text{add1}}(\vec{n}_0, \vec{r}_0) + \vec{E}_d(\vec{n}_0, \vec{r}_0) + \\ &+ \vec{E}_{\perp}^{\text{bas}}(\vec{n}_0, \vec{r}_0) + \vec{E}_{\perp}^{\text{add1}}(\vec{n}_0, \vec{r}_0). \end{aligned} \quad (5)$$

In (5), the first two terms represent the information component decomposition of the receiving antenna scattering field, the last two terms are the decomposition of residual component of the receiving antenna scattering field, and the middle term represents the scattering field diagram component, which arises due to the mismatch between the antenna and the load ($\tilde{A} \neq \gamma$).

Such a detailed decomposition of the scattered field made it possible to conduct a detailed study of the interaction process of each scattered field component with the plane wave field, but led to a system of four optical theorems [7-9] for each interaction channel in the form

$$-\frac{2\pi}{kZ_0} \text{Im} \left(\vec{e}_0, \vec{A}_{\text{inf}}^*(\vec{n}_0, \vec{n}_0) \right) = P_L + P_d + P_{\text{inf}} = 2P_L^{\text{max}}, \quad (6)$$

$$-\frac{2\pi}{kZ_0} \text{Im} \left(\vec{e}_0, \vec{A}_d^*(\vec{n}_0, \vec{n}_0) \right) = -2P_L^{\text{max}} \text{Re} \left(\frac{\tilde{A} - \gamma}{1 - \gamma\tilde{A}} \vec{A}_{\perp}^* \right) = P_{\text{mut}}, \quad (7)$$

$$-\frac{2\pi}{kZ_0} \text{Im} \left(\vec{e}_0, \vec{A}_{\text{inf}}^{\text{add1}}(\vec{n}_0, \vec{n}_0) \right) = \alpha_2 \frac{\pi D}{k^2 Z_0} \left| (\vec{F}(-\vec{n}_0) \cdot \vec{e}_0) \right|^2, \quad (8)$$

$$-\frac{2\pi}{kZ_0} \text{Im} \left(\vec{e}_0, \vec{A}_{\perp}^*(\vec{n}_0, \vec{n}_0) \right) = P_{\perp}^{\text{bas}} + P_{\perp}^{\text{add1}} + P_{\perp}^{\text{mut}} + P_{\text{inf}}^{\text{add1}}. \quad (9)$$

Moreover, in [5] it was assumed that the receiving antenna has no losses. Therefore, it is not possible to directly use these results to solve the problem of taking into account the influence of losses in the operation of interaction channels antenna. In what follows, we will be mainly interested in case of a matched receiving antenna with its load $\tilde{A} = \gamma$. When this condition is met, the controlled diagram component of stray field is zero. Taking this into account, expression (5) takes the form

$$\vec{E}_s(\vec{n}_0, \vec{r}_0) = \vec{E}_{\text{inf}}^{\text{bas}}(\vec{n}_0, \vec{r}_0) + \vec{E}_{\text{inf}}^{\text{add1}}(\vec{n}_0, \vec{r}_0) + \vec{E}_{\perp}^{\text{bas}}(\vec{n}_0, \vec{r}_0) + \vec{E}_{\perp}^{\text{add1}}(\vec{n}_0, \vec{r}_0). \quad (10)$$

Since in this work it is mainly important to study the influence of losses in the antenna on the operation of the interaction channel through which information is transmitted from a plane wave to the load, it is reasonable at the initial stage to combine the components of the information component of the scattering field of the receiving antenna:

$$(\vec{E}_{\text{inf}}^{\text{bas}}(\vec{n}_0, \vec{r}_0) + \vec{E}_{\text{inf}}^{\text{add1}}(\vec{n}_0, \vec{r}_0) = \vec{E}_{\text{inf}}), \text{ and residual component:}$$

$$(\vec{E}_{\perp}^{\text{bas}}(\vec{n}_0, \vec{r}_0) + \vec{E}_{\perp}^{\text{add1}}(\vec{n}_0, \vec{r}_0) = \vec{E}_{\text{res}}).$$

Taking these remarks into account, the receiving antenna scattering field will be further considered as the sum of a single information component of the receiving antenna scattering field and a single residual component in the form

$$\vec{E}_s(\vec{n}_0, \vec{r}_0) = \vec{E}_{\text{inf}} + \vec{E}_{\text{res}} \quad (11)$$

This approach will make it possible to move from a four-channel model to a two-channel model, which greatly simplifies further analysis of the results obtained. Thus, the electric and magnetic fields existing around the receiving antenna, taking into account (11), can be written in the form

$$\vec{E}(\vec{n}_0, \vec{r}_0) = \vec{E}^i(\vec{n}_0, \vec{r}_0) + \vec{E}_{\text{inf}} + \vec{E}_{\text{res}}, \quad (12)$$

$$\vec{H}(\vec{n}_0, \vec{r}_0) = \vec{H}^i(\vec{n}_0, \vec{r}_0) + \vec{H}_{\text{inf}} + \vec{H}_{\text{res}}. \quad (13)$$

Let us consider the volume V in which the receiving antenna under study is located. This volume is limited by the surface S , which can be considered as the sum of three surfaces S_1 , S_2 and S_3 . S_1 is the cross-sectional surface of the antenna feeders through which the power received by the antenna enters the load. S_3 is the surface of the receiving antenna, through which the power of the incident wave enters the walls of the antenna and is converted into heat. S_2 is the surface of a sphere of infinite radius at the center on which the antenna is located.

Fields (12) and (13) inside volume V satisfy Maxwell's equations

$$\text{rot}(\vec{H}) = i\omega\varepsilon_0 \vec{E} + \vec{j}^{\text{st}}, \quad (14)$$

$$\text{rot}(\vec{E}) = -i\omega\mu_0 \vec{H}, \quad (15)$$

where \vec{j}^{st} – external current density. Considering that in volume V under consideration there are no third-party sources, for the complex conjugate quantities included in (14), relation is valid

$$\text{rot}(\vec{H}^*) = -i\omega\varepsilon_0 \vec{E}^*. \quad (16)$$

Multiplying scalarly both sides of (16) by the vector $\dot{\vec{E}}$ taking into account (15), expansions (12), (13), the known relation $div[\dot{\vec{E}}, \dot{\vec{H}}] = \dot{\vec{H}} rot(\dot{\vec{E}}) - \dot{\vec{E}} rot(\dot{\vec{H}})$ and omitting a number of known transformations, it is easy to obtain the following energy relationship, which relates power released in receiving antenna load P_L and scattering power of the antenna scattering field information component P_{inf} with the power lost in the antenna due to its finite conductivity P_a and scattering power of the antenna scattering field residual component P_{res}

$$P_L + P_{inf} - P_{inf}^i = -P_a - P_{res}^{inf} + P_{res}^i - P_{res} \quad (17)$$

In (17) P_{res}^{inf} – mutual power between the information and antenna scattering field residual components and P_{inf}^i and P_{res}^i – extinction powers taken from a plane wave through the information and residual interaction channels, respectively. These quantities are generally determined by the following relations

$$P_{inf}^i = \frac{1}{Z_0} \oint_{s_2} \dot{\vec{E}}^i(\vec{n}_0, \vec{r}_0) \dot{\vec{E}}_{inf}^*(\vec{n}_0, \vec{r}_0) \partial \vec{S}, \quad (18)$$

$$P_{res}^i = \frac{1}{Z_0} \oint_{s_2} \dot{\vec{E}}^i(\vec{n}_0, \vec{r}_0) \dot{\vec{E}}_{res}^*(\vec{n}_0, \vec{r}_0) \partial \vec{S}, \quad (19)$$

$$P_{res}^{inf} = \frac{1}{Z_0} \oint_{s_2} \dot{\vec{E}}_{inf}(\vec{n}_0, \vec{r}_0) \dot{\vec{E}}_{res}^*(\vec{n}_0, \vec{r}_0) \partial \vec{S}. \quad (20)$$

To calculate integrals of type (18) and (19), the stationary phase method is usually used [8]. Taking this into account, (18) and (19) can be written in the form

$$P_{inf}^i = -\frac{2\pi}{kZ_0} Jm \left(\dot{\vec{e}}_0, \dot{\vec{A}}_{inf}(\vec{n}_0, \vec{n}_0) \right), \quad (21)$$

$$P_{res}^i = -\frac{2\pi}{kZ_0} Jm \left(\dot{\vec{e}}_0, \dot{\vec{A}}_{res}(\vec{n}_0, \vec{n}_0) \right), \quad (22)$$

where $\dot{\vec{A}}_{inf}(\vec{n}_0, \vec{n}_0)$ and $\dot{\vec{A}}_{res}(\vec{n}_0, \vec{n}_0)$ – values of scattering patterns of the information and residual components of the antenna scattering field in wave propagation direction \vec{n}_0 .

In case of matched load ($\tilde{A} = \gamma$) the power released in it is maximum and is determined by the ratio

$$P_L^{\max} = \frac{\pi G}{2k^2 Z_0} \left| (\dot{\vec{F}}(-\vec{n}_0) \cdot \dot{\vec{e}}_0) \right|^2, \quad (23)$$

In the absence of losses in the antenna, the information component of stray field involved in transfer of power to the load has the form [1]

$$\vec{E}_{\text{inf}}(\vec{n}_0, \vec{r}_0) = \frac{D}{2ik} (\dot{\vec{F}}(-\vec{n}_0) \cdot \dot{\vec{e}}_0) \dot{\vec{F}}^*(-\vec{r}_0) \frac{\exp(-ikr)}{r}. \quad (24)$$

From physical considerations it is clear that the appearance of losses in the antenna will lead to a change in the law of current distribution on its surface, and this in turn will affect the shape of radiation pattern and antenna maximum efficiency. In addition, the antenna gain will change. Taking these considerations into account, the expression for

$\vec{E}_{\text{inf}}(\vec{n}_0, \vec{r}_0)$ can be written as

$$\vec{E}_{\text{inf}}(\vec{n}_0, \vec{r}_0) = \frac{G}{2ik} (\dot{\vec{F}}(-\vec{n}_0) \cdot \dot{\vec{e}}_0) \dot{\vec{F}}^*(-\vec{r}_0) \frac{\exp(-ikr)}{r}. \quad (25)$$

In (25) $\dot{\vec{F}}(\vec{r}_0)$ – This is the antenna pattern taking into account losses.

Moreover

$$\dot{A}_{\text{inf}}(\vec{n}_0, \vec{n}_0) = \frac{G}{2ik} (\dot{\vec{F}}(-\vec{n}_0) \cdot \dot{\vec{e}}_0) \dot{\vec{F}}^*(-\vec{n}_0) \quad (26)$$

Substituting (26) into (21) we determine the extinction power taken from a plane wave through the information interaction channel

$$P_{\text{inf}}^i = \frac{\pi G}{k^2 Z_0} \left| (\dot{\vec{F}}(-\vec{n}_0) \cdot \dot{\vec{e}}_0) \right|^2 = 2P_L^{\text{max}}. \quad (27)$$

The scattering power of the antenna scattering field information component can be determined from the relation

$$P_{\text{inf}} = \frac{1}{2Z_0} \oint_{s_2} \left| \vec{E}_{\text{inf}}(\vec{n}_0, \vec{r}_0) \right|^2 \partial \vec{S}. \quad (28)$$

Substituting expression (26) into (28) for P_{inf} we get

$$P_{\text{inf}} = \frac{\pi G \eta}{2k^2 Z_0} \left| (\dot{\vec{F}}(-\vec{n}_0) \cdot \dot{\vec{e}}_0) \right|^2 = \eta P_L^{\text{max}}. \quad (29)$$

Taking into account (27) and (29), expression (17) leads to two optical theorems.

$$P_a + P_{\text{res}}^{\text{inf}} + P_{\text{res}} = P_L^{\text{max}} (1 - \eta) + P_{\text{res}}^i \quad (30)$$

$$P_L + P_{\text{inf}} = P_L^{\text{max}} (1 + \eta), \quad (31)$$

The first of them (30) describes the joint work of residual and information channels of interaction, and second (31) of the information channel of interaction.

Finally, if the right-hand sides of (30) and (31) are expressed in terms of the extinction powers of the information and residual interaction channels, then the optical theorems for these interaction channels take the form

$$P_L + P_{\text{inf}} = \left(\frac{1+\eta}{2} \right) P_{\text{inf}}^i \quad (32)$$

$$P_a + P_{\text{res}}^{\text{inf}} + P_{\text{res}} = \left(\frac{1-\eta}{2} \right) P_{\text{inf}}^i + P_{\text{res}}^i \quad (33)$$

Thus, problem posed at the beginning of work about studying the antenna losses influence, which are characterized by the antenna efficiency, on the interaction information channel operation η has been completely solved, which allows us to draw the following conclusions. In contrast to the case of ideally conducting walls of the receiving antenna, losses lead to the fact that not all the power taken through the information interaction channel from a plane wave is spent on the power released in the load and the power of the information component of the scattering field of the receiving antenna, but only part of it, determined expression $\left(\frac{1+\eta}{2} \right) P_{\text{inf}}^i$.

The remaining part is equal to $\left(\frac{1-\eta}{2} \right) P_{\text{inf}}^i$, is spent on heat loss power along with the power transmitted through the residual interaction channel. As the losses in the receiving antenna decrease, the antenna η efficiency tends to unity and the expressions obtained in this work transform into the expressions we obtained earlier for the case of lossless receiving antennas.

3 Conclusion

In this work, investigated the influence of thermal losses in the walls of the receiving antenna on the energy of interaction of the components of the scattering field of the receiving antennas with the field of the incident plane wave. As a research method, we used the multichannel theory of receiving antennas, the effectiveness of which has been repeatedly demonstrated by us in solving problems of analyzing ideally conducting receiving antennas.

As a result of the study, a two-channel model was built, which made it possible to study very subtle relationships between the powers of the scattering field components of the receiving antenna, the power released in the antenna load and the power lost in the antenna walls due to their finite conductivity.

As a basis for the decomposition of the total scattering field of the antenna, we used the representation of the field as the sum of two components, one of which is involved in the transfer of energy to the antenna load and (information component of the scattering field of the receiving antenna) the remaining part (residual component of the receiving antenna's stray field), which is not involved in the transfer of power from a plane wave to the antenna load. The two-channel model completely solved the problem posed above in analytical form, which made it possible to draw conclusions about the influence of losses in the walls of the receiving antenna on the energy of the information interaction channel without conducting numerical studies.

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