COMPUTATIONAL FEATURES OF CALCULATING EXTENDED CONDUCTOR ANTENNAS BASED ON THE INTEGRAL EQUATIONS METHOD

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ABSTRACT

The numerical solution of integral equations or a system of integral equations (for a system of conductors) consists in their discretization and reduction to a system of linear algebraic equations for the desired function. For discretization, it is possible to use projection methods, for example, the Galerkin method or the collocation method. A system of linear algebraic equations for the class of problems under consideration is characterized by a complete (filled) complex matrix. For conductor antenna structures that are quite arbitrary in geometry and size, systems of linear algebraic equations turn out to be of a high order, and special computational algorithms are required to solve them. The purpose of the work is to study adequate mathematical models for wire antennas with a subsequent description of uniform numerical algorithms based on methods of sampling and approximation of antenna current. With this method, the error in solving integral equations is determined by the error in calculating the elements of the matrix of a system of linear algebraic equations for a given piecewise polynomial approximation of the solution at step h. If the quadrature formula for numerical integration is chosen, then there are two ways to reduce the solution error. An increase in the number of collocation points leads to a rapid increase in the volume of calculations and, consequently, to a rapid increase in the amount of occupied computer memory. Therefore, the question arises of choosing the most profitable method of piecewise polynomial interpolation and discretization step h from the point of view of using computer resources, ensuring an acceptable number of solutions.

KEYWORDS: *Integral Equations, Mathematical Modeling, Conductive Antennas.*

1 Introduction

Wire dipole antennas, especially as part of arrays, are the most common type of modern antennas for various purposes and wave ranges. With the development and complication of electronic equipment, the problem of designing wire antennas arises, especially for modern communication systems, which have design and technological advantages.

The basis of mathematical modeling and the creation of computational algorithms for the analysis of these antennas is the reduction of the corresponding boundary value problems of electrodynamics to integral equations for the antenna current. Integral equations are valid for the entire frequency range, have a smaller dimension than the boundary value problem, and are universal with respect to the geometry of the antenna conductors. From a computational point of view, the most effective seems to be the use of integral and integro-differential equations of the first kind, which lead to the construction of uniform and efficient algorithms for numerical analysis of antennas. If the antenna current is known, then calculating its characteristics does not pose any fundamental difficulties.

The numerical solution of integral equations or a system of integral equations (for a system of conductors) consists in their discretization and reduction to a system of linear algebraic equations for the desired function. For discretization, it is possible to use projection methods, for example, the Galerkin method [1,2] or the collocation method. A system of linear algebraic equations for the class of problems under consideration is characterized by a complete (filled) complex matrix. For conductor antenna structures that are quite arbitrary in geometry and size, systems of linear algebraic equations turn out to be of a high order, and special computational algorithms are required to solve them.

The purpose of the work is to study adequate mathematical models for wire antennas with a subsequent description of uniform numerical algorithms based on methods of sampling and approximation of antenna current [3].

2 Technique modelling

The key problem can be identified as the problem of excitation (diffraction) on a thin wire of arbitrary geometry. A curved wire antenna (Fig. 1) with wire size $ka \ll 1$, $k = 2\pi/\lambda$, λ is the operating wavelength, will be characterized by a generatrix in the form of a piecewise smooth curve ζ. The geometry of the latter is described by an orthogonal curvilinear coordinate system (s, v) with Lamé coefficients $h1$, $h2$. Let us imagine the antenna input with a slot of size tb, $kb \ll 1$. With a potential difference U of

At the entrance to the slot, the primary field $E0$ is established.

For the current $I(l)$ of a wire antenna, we can obtain

$$\frac{\partial}{\partial l}\left\{\int_{\zeta}(S^0,S_0^0)l(l_0)\frac{\partial}{\partial l_0}G(M,M_0)dl_0\right\}-$$
$$-k^2\int_{\zeta}(S^0,S_0^0)l(l_0)\frac{\partial}{\partial l_0}G(M,M_0)d=$$
$$=-4i\pi\omega\varepsilon(E^0,S^0) \tag{1}$$

where (S^0,S^0), (E^0,S^0) –scalar products of unit vectors at points M and M_0 (Fig. 1), $dl = h1\cdot ds$ – elements of length.

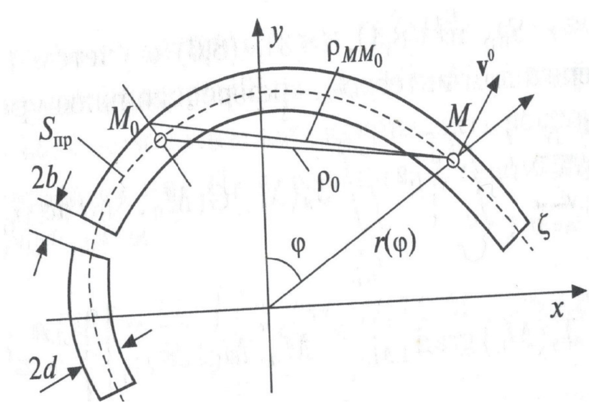

Fig. 1: Curved wire antenna

Equation (1) is an integro-differential equation for current *I*(*l*) of a wire antenna. In the excitation problem, *E0* = *U*/(2*b*), where U is the potential difference at the antenna input. Equation (1) is known as the Poe-Clington integral equation.

For the case of curves ζ of constant curvature h^1, h^2 = const, isolating the differential operator (d^2 / $d\,l^2$ + k^2) and using the well-known inversion formula, we obtain

$$\int_{\zeta} I(l_0)K(l,l_0)dl_0 = i\frac{k}{2\omega}\int_{\zeta} E_0(u)\sin(k\,|\,l-u\,|\,du + c_1\sin kl + c_2\cos kl, \tag{2}$$

where the equation kernel has the form

$$K(l,l_0) = (S^0, S_0^0)G(M, M_0)$$
$$G(M, M_0) = \frac{\exp(-ik\sqrt{a^2+(l-l')^2})}{a^2+(l-l')^2}, \tag{3}$$

a – wire radius, *M*, M_0 – points on the curve ζ. Coefficients c_1 and c_2 are selected from additional conditions (current equal to zero at the ends of the wire). Equation (2) is known as the Gallen integral equation.

Integral equation (3) is used to analyze conductive structures of constant curvature, which include linear, ring and equiangular structures.

Integral equation (1) and (2) are Fredholm equations of the first kind. Their use does not provide great advantages, however, the Gallen equation is more general in terms of specifying the field on the right side of the equation created by the excitation source [4-6]. For wire antenna currents we have integro-differential or integral equations of the first kind. The numerical solution of these equations involves their discretization and approximation of the current and reduction to a system of linear algebraic equations. Solving equations of the first kind is, in the general case, an ill-posed problem according to A. N. Tikhonov due to the numerical instability of the solution (depending on the right side of the equation, which determines the method of excitation of the antenna) and requires regularization. Regularization methods and algorithms can be different. For the integral equations under consideration, the most convenient is a simple regularization method, which is called the self-regularization method [7]. The method uses the a priori property of smoothness of the desired solution and the weak singularity of the kernels of integral equations.

Let us consider the integro-differential equation (1) for the wire antenna current. The algorithm that implements the principle of self-regularization of an equation consists of identifying the singularity of its kernels, local interpolation of the desired solution and its calculation in a given set of collocation points. Let us determine the largest size of the wire structure Spr along the contour ζ as $2L$ and divide it into N segments with a step $h = 2L/N$. For the ends of the partition segments and nodes of the segments, we determine the values

$$l_{i-1/2} = -Lh\left(i - \frac{3}{2}\right);\ l_{i+\frac{1}{2}} = -\left(i - \frac{1}{2}\right);\ l_i = -Lh(i-1), i = 2,...,N,$$

We assume a piecewise constant approximation of the current and at the nodes we determine the coefficients $I(l_i) = I_i$. at collocation points we obtain from the system of linear algebraic equations

$$\sum_{i=2}^{N} I_i A_{i,j} = F_j,\ \ j = 2,...,N, \tag{4}$$

where

$$A_{ij} = D_{i,j+1/2} - D_{i,j+1/2} - B_{i,j}$$

$$D_{j,i+1/2} = \frac{\partial}{\partial l} G(l, l_{i+1/2}\ |_{l=l_j};\ D_{j,i-1/2} = \frac{\partial}{\partial l} G(l, l_{i-1/2}\ |_{l=l_j}$$

$$B_{ji} = \int_{l_{i-1/2}}^{l_{i+1/2}} (S_j, S^0) G_0(l_j - l_0) dl_0$$

$$F_j = \begin{cases} (E^0, S^0), j = \dfrac{N}{2} + 1 \\ 0,\ \ j \neq \dfrac{N}{2} + 1. \end{cases}$$

In (4), the conditions at the ends of conductor $I1,...,IN+1 = 0$ are taken into account. At a sufficiently small value of h, the diagonal elements of the system of linear algebraic equations exceed in absolute value the remaining elements of the matrix, which ensures its stable solution. Sampling step $h < 0.05\lambda$.

Let's consider the integral equation (2). The algorithm for its numerical solution, which implements the principle of self-regularization, consists of isolating the singularity of the kernel, local interpolation of the desired solution and reducing it when discretized with a step h (the distance between neighboring collocation points) to a well-conditioned system of linear algebraic equations. In contrast to the algorithm discussed above, it is assumed that the collocation points coincide with the interpolation nodes.

Let us set the conductor to be divided into N parts with a step $h = 2L/N$ and form a grid of variables $\{l_i, l_j\}$, i, j = 1,…,N+1. For example, assuming a piecewise constant approximation of the current $I(l_i) = I_i$, at the sampling step h from (2) we obtain a system of linear algebraic equations

$$\sum_{i=2}^{N} A_{i,j} I_i = B_j + c_1 \sin l_j +\ c_2 \cos l_j,\ \ j = 1,...,N, \tag{5}$$

where

$$A_{ij} = \int_{l_{i-1/2}}^{l_{i+1/2}} K(l_j, l_0) dl_0$$

$$B_j = \int_{-L}^{L} F^0(u) \sin|l_j - u| du$$

$$l_{i-1/2} = h(i-3/2) - L; \; l_{i+1/2} = h(i-1/2) - L.$$

System (5) takes into account the condition at the ends of conductor $I(L) = I(-L) = 0$. In this case, the matrix is further defined by the elements Aj,1 = -coslj, Aj, N + 1 = -sinlj, j = 1,…, N + 1, and the coefficients c_1 and c_2 are among the unknowns of the system. The matrix of system (5), due to the distinguished feature of the kernel, has a diagonal predominance and the solution of the system is stable. The regularization parameter is step h < 0.05λ.

We will consider the features of constructing a numerical solution of integral equations of the 1st kind using the example of solving integral equations for the total current of a strip vibrator

$$\int_{-L}^{L} l(x_0) G(x, x_0) dx_0 = \frac{U}{60} J_0(b) \sin|x| + C_1 \sin x + C_2 cosx, \quad (6)$$

where 2L is the length of vibrator, 2b is the size of slot at its input, U is the potential difference at the input, $G(x,x_0)$ is the kernel of the quation, which has a logarithmic feature, C_1, C_2 are coefficients determined from additional conditions at the ends of vibrator.

The permissible values of the sampling step with an error in calculating the vibrator current of no more than 5% are given in Table 1.

Table 1. Sampling step

Type of current approximation	h	h(λ)
Piecewise constant	1,16	0,026
Piecewise trigonometric	0,262	0,042
Piecewise quadratic	0,657	0,105
Spline	0,98	0,156

3 Conclusion

Stability of the solution to a system of linear algebraic equations in the self-regularization algorithm is ensured by the system matrix diagonal elements predominance.

The criterion for correctness of an approximate solution is its internal convergence in a series of calculations when the collocation points are condensed (increasing the order N of the matrix) with subsequent estimation of the discrepancy of the solution.

With this method, the error in solving integral equations is determined by the error in calculating the elements of the matrix of a system of linear algebraic equations for a given piecewise polynomial approximation of the solution at step h. If the quadrature formula for numerical integration is chosen, then there are two ways to reduce the solution error.

The first method is that for a given degree of the interpolation polynomial, the sampling step h is reduced.

The second method is that while maintaining the step h, the degree of the polynomial increases, i.e. the number of interpolation nodes used for this increases. However, the degree of the approximating polynomial should not be too large, since this significantly complicates the software implementation of the algorithm and increases the time it takes to calculate the matrix of a system of linear algebraic equations on a computer.

An increase in the number of collocation points leads to a rapid increase in the volume of calculations and, consequently, to a rapid increase in the amount of occupied computer memory. Therefore, the question arises about choosing the most profitable method of piecewise polynomial interpolation and discretization step h from the point of view of using computer resources, ensuring an acceptable number of solutions [8].

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