# THE EFFICIENCY RESEARCH OF MIMO SYSTEMS WITH CORRELATED ELEMENTS OF A CHANNEL TRANSMISSION MATRIX

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**ABSTRACT**

Currently, there is an active development of wireless data transmission systems. A solution widely used in improving existing and creating new wireless communication technologies, which allows increasing spectral efficiency and transmission speed, is the use of systems with Multiple-Input-Multiple-Output, MIMO. In this regard, research into the potential capabilities of MIMO systems under various operating conditions is an urgent task. In particular, the issue of the influence of the correlation of transmission matrix elements on the efficiency of the communication system is important. We will further consider cases of multipath propagation typical for urban conditions. We will also consider the signals transmitted by different antennas to be narrowband and independent of each other. For the two currently most commonly used MIMO radio channel models, the influence of the correlation between the elements of the transmission channel matrix on the efficiency of the MIMO system was studied using statistical tests. Dependences of the error probability per bit of transmitted information on the signal-to-noise ratio were obtained for channels with transmission matrices whose covariance matrices have varying degrees of difference from the diagonal (unit) matrix. When generating channel matrices with correlated elements, the unstructured (general correlation) model and the Kronecker model are used.

**KEYWORDS:** *MIMO, Transmission Channel Matrix, Monte Carlo Method.*

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## 1 Introduction

A solution widely used in improving existing and creating new wireless communication technologies to increase spectral efficiency and transmission speed is the use of systems with *nt* transmitting and *nr* receiving antennas (Multiple-Input-Multiple-Output, MIMO). In this regard, studying the potential capabilities of MIMO systems under various operating conditions is an urgent task. In particular, an important issue is the influence of the correlation of transmission matrix elements on the efficiency of the communication system [1-4].

We will further consider cases of multipath propagation typical for urban conditions. We will also consider the signals transmitted by different antennas to be narrowband and independent of each other.

## 2 Research methods

Let us assume that a narrowband signal $s_k(t)$ with a complex envelope $x_k(t)$ is supplied to the kth element of the transmitting antenna array. This element emits an electromagnetic wave, which, when propagating from the transmitter to the receiver, takes many complex paths with reflections and diffraction from obstacles. As a result, the *i*-th element of the receiving antenna array is affected by a large number of electromagnetic waves, each of which creates a narrow-band signal in this element $s_{ik}^l(t)$ complex envelope $a_{ik}^l(t) = h_{ik}^l x_k(t-\tau)$, where *l* – received wave number, $h_{ik}^l$ – complex transmission coefficient of the channel along the *l*-th path, $\tau$ – time of wave propagation from the transmitter to the receiver. For the complex envelope of the total signal of this element, we can write:

$$a_{ik}(t) = \sum_{l=1}^{L} h_{ik}^l x_k(t-\tau) = h_{ik} x_k(t-\tau). \quad (1)$$

where $h_{ik}$ – equivalent complex channel transmission coefficient from the *k*-th element of the transmitting antenna to the *i*-th element of the receiving antenna; *L* – number of summed waves.

The model of the received signal (1) is based on the following assumptions:

1) for the equivalent channel transmission coefficient the following representation is valid:

$$h_{ik} = \sum_l Re(h_{ik}^l) + j \cdot \sum_l Im(h_{ik}^l); \quad (2)$$

2) all terms in (1) have the same delay time $\tau$, i.e. the propagation time of the corresponding electromagnetic waves is the same, although each of them “passed” along its own path, perhaps significantly different from the others;

3) the channel transmission coefficients, at least on a local time interval, have constant values (channel with constant parameters).

If these conditions are met, then it can be shown that the following statements are true:

4) channel transmission coefficients hik are independent complex Gaussian random variables with zero mean and independent real and imaginary parts with variances $\sigma_{ik}^2$;

5) random riable $|h_{ik}|$ has a Rayleigh distribution with parameter $\sigma_{ik}$.

For simplicity, we will further assume that $E\{h_{ik}h_{ik}^*\} = 1$.

The channel gains introduced above are usually written as a matrix of size $n_r$ x $n_t$:

$$\boldsymbol{H} = \begin{bmatrix} h_{11} & \cdots & h_{1n_t} \\ \vdots & \ddots & \vdots \\ h_{n_r1} & \cdots & h_{n_rn_t} \end{bmatrix}. \quad (3)$$

If we introduce the following notations: $x = (x^l\ x_2\ ...\ x_{nt})^T$ – for the vector of samples of complex signal envelopes at the input of the elements of the transmitting antenna array, $y = (y_l\ y_2\ ...\ y_{nr})^T$ – for the vector of samples of complex envelopes of signals at the output of the elements receiving antenna array, then we can write

$$y = Hx + n, \tag{4}$$

where $n = (n_l\ n_2\ \dots\ n_{nr})^T$ – ector of the noise envelope complex samples.

Most of the results of modern research on MIMO systems are obtained for models which turn out to be adequate to the above-presented conditions for the functioning of these systems [5,6,7]. Recently, quite a lot of works have been published that consider various models of the channel matrix H with correlated elements [6, 8, 9]. In this work, some of these models are used to evaluate the noise immunity of MIMO systems. The issue of the influence of correlation has already been considered by the authors in [10]. However, in this work, a different correlation criterion is used, and a standardized model is used to obtain covariance matrices.

### 3 Models of MIMO systems with correlated elements of transmission matrices

The efficiency of MIMO systems depends on the channel saturation with independent radio wave propagation paths from the transmitter to the receiver. An indicator of this indicator is correlation between the elements of transmission matrix. The presence of multiple propagation paths leads to a lack of correlation, which indicates the high efficiency of the MIMO system. However, the model with uncorrelated elements is only a special case of possible scenarios. Here we will consider a model with unstructured correlations, which we will call the general correlation model, and the Kronecker model.

Let us introduce a matrix of size correlation coefficients $n_r n_t$ x $n_r n_t$ [5]

$$R = E[\mathrm{vec}(H)\mathrm{vec}(H)^H], \tag{5}$$

where vec($H$) is an operator mapping the matrix $H$ into the vector vec(H) = $[h_{ll} \dots h_{nrl}\ h_{l2} \dots h_{nr2} \dots h_{nrnt}]^T$.

The matrix $R$ is Hermitian, positive semidefinite and completely describes the statistical properties of the channel, since the elements $h_{ik}$ have a joint normal distribution with zero mean values.

The implementation of channel $H$ with a given matrix $R$ can be obtained using the formula:

$$\boldsymbol{H} = unvec\left(\boldsymbol{R}^{\frac{1}{2}} vec(\boldsymbol{H_w})\right), \tag{6}$$

where $H_w$ is a matrix with uncorrelated elements; $R^{1/2} = UD^{1/2}U^H$; $UDU^H = R$ – spectral (in terms of eigenvalues) decomposition of a symmetric positive semidefinite matrix.

To simplify the problem being solved, additional restrictions are often introduced. The most widely used model in the literature is the Kronecker product model, which introduces the following assumptions [11]:

the correlation moments "at reception" $E[h_{ij}h_{kj}^*]$ and "at transmission" $E[h_{mj}\ h_{mk}]$ do not depend on $j$ and $m$, respectively; this means that the correlation of the transmission coefficients depends only on the selected pair of antennas on the corresponding side.

Mutual covariance moments can be represented as $E[h_{ij}h_{km}] = E[h_{ij}h_{kj}][h_{im}h_{km}]$; this means that the "joint" covariance moments are defined as the product of the corresponding "receive" and "transmit" correlations.

This structure of correlation coefficients is valid for scenarios where objects locally distributed around the transmitter are sufficiently separated from objects surrounding the receiver. If these conditions are met, then the matrix R can be represented as a Kronecker product [11]:

$$\boldsymbol{R} = \boldsymbol{R_t} \otimes \boldsymbol{R_r}, \quad (7)$$

where $R_t = E([H^H H])^T$, $R_r = E[HH^H]$ are the "transmit" and "receive" correlation matrices, respectively.

For the Kronecker product model, the matrix of channel transmission coefficients with correlated elements can be represented as a linear transformation of a matrix with uncorrelated coefficients:

$$\boldsymbol{H}_{kron} = \boldsymbol{R}_r^{1/2} \boldsymbol{H}_w \boldsymbol{R}_t^{T/2}. \quad (8)$$

This representation allows us to significantly simplify the analysis of the effectiveness of MIMO technology in conditions of correlation of channel matrix elements H. The Kronecker model, due to its simplicity, is used in many works devoted to the study of the impact of spatial correlation on the quality of a MIMO system. However, this model also has some disadvantages [5].

Numerical values of covariance moments can be obtained empirically [5]; There are also works in which these moments can be obtained on the basis of geometric models of radio wave propagation [12-15].

For modeling in this work, the recommended 3GPP MIMO channel model was used [16]. This model is geometric (Fig. 1) and allows us to obtain numerical values of transmission matrices for scenarios of urban macro- and microcells. The resulting matrices contain normally distributed complex quantities and have covariance matrices $R_{macro}$ and $R_{micro}$

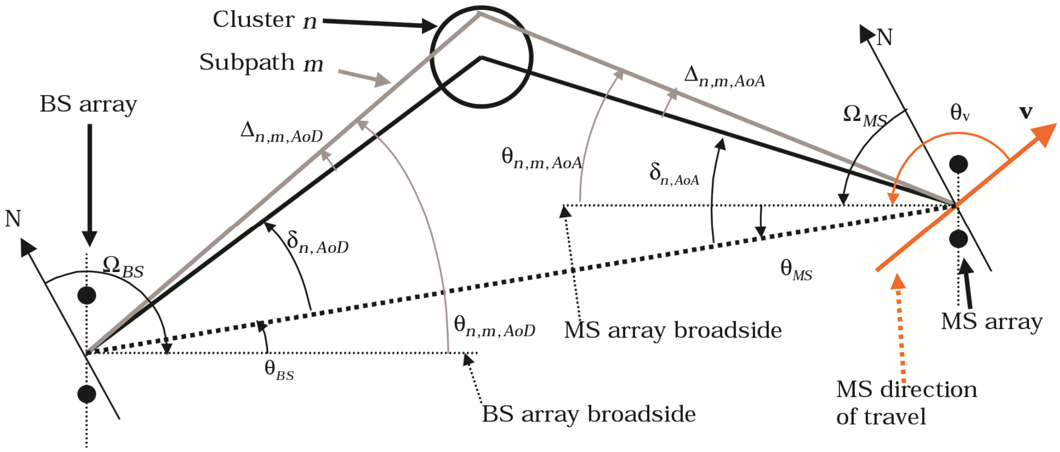


**Fig. 1:** BS and MS angle parameters [16]

The saturation of a channel with multipath propagation paths is determined through a measure of effective diversity [11]:

$$N_{div} = \frac{(\sum \lambda_k(R))^2}{\sum {\lambda_k}^2(R)}, \quad (9)$$

where $\lambda_k(R)$ are the eigenvalues of the covariance matrix $R$.

The measure $N_{div}$ is maximum for an uncorrelated channel ($R = I$) and is equal to $n_r\, n_t$, and for a fully correlated channel ($R = 1$) it is equal to unity, where

$$\mathbf{1} = \begin{pmatrix} 1 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & 1 \end{pmatrix}$$

and ***I*** — identity matrix. It can be shown [11] that the value of Ndiv determines the rate at which the error probability decreases with increasing SNR for channels with Rayleigh fading.

### 4 Noise immunity of MIMO systems with correlated channel matrix elements

Using the Monte Carlo method, error probability plots were obtained for the above 8x8 MIMO channel correlation models for a system with PM2 modulation and maximum likelihood (ML) reception.

Transmission in the model was carried out using spatial multiplexing (SM) mode with uniform power distribution between the antennas. The estimate of the transmission matrix was assumed to be known at the receiving end and unknown at the transmitting end (open-loop system).

The case of the general correlation model and the case of the Kronecker product model were considered. The transfer matrices under consideration are the matrix

$$tr\{R\} = n_t n_r.$$

The simulation results are presented in Figure 2.

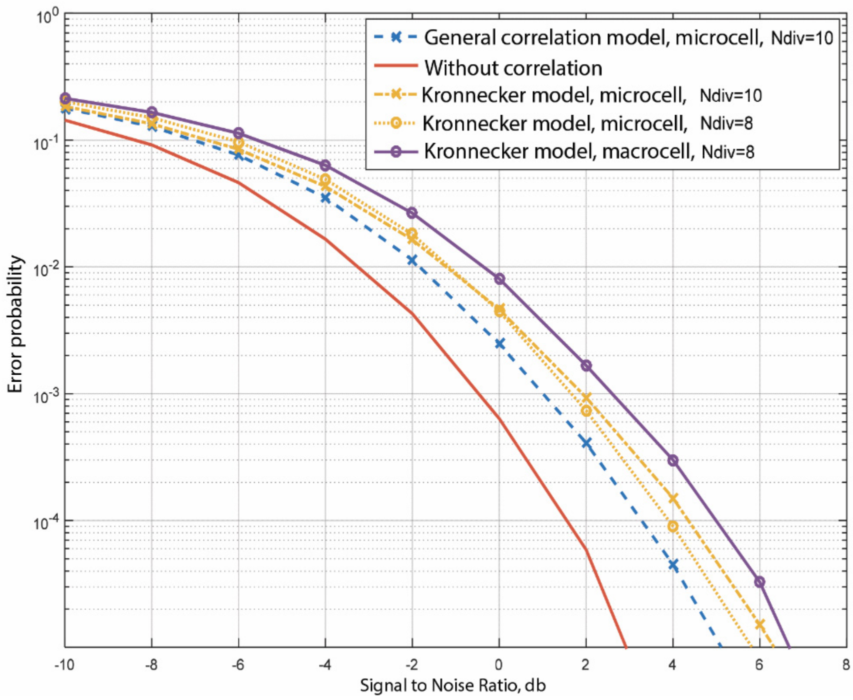


**Fig. 2:** Plot of error probability versus signal-to-noise ratio for SM-MIMO-ML 8x8 system

### 5 Conclusion

The elements of the channel gain matrix are usually considered to be independent complex random variables with a joint normal distribution. In general, these elements are correlated with each other, which is true for most real-life scenarios. The results presented here indicate that the presence of correlation significantly affects the performance of a MIMO system and, in general, as correlation increases, the noise immunity of MIMO decreases noticeably. It should also be noted that the Kronecker product model, which is most often used in the literature, although it simplifies the task of analyzing the influence of correlation, however, leads to inflated estimates of losses.

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