STATISTICAL CHARACTERISTICS OF TWO-DIMENSIONAL AUTOCORRELATION FUNCTIONS IN NOISE-LIKE COMPLEX SIGNALS

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ABSTRACT

For various terrestrial and satellite radio systems, the task of detecting complex signals under conditions of a priori uncertainty, when their parameters, including the modulation law, is unknown, is relevant. Detection of such signals can be combined with estimation (measurement) of their parameters, such as carrier frequency and delay time. A method for calculating two-dimensional autocorrelation functions of noise-like complex signals has been developed and their types have been studied depending on the type of pseudo-random sequences on the basis of which they are formed, as well as the modes of emission of these signals in radio systems. Regularities have been revealed in the statistical characteristics of side peaks of various types of complex signals depending on the length of pseudo-random sequences and radiation modes. The developed methodology for calculating the two-dimensional correlation function of complex signals made it possible to identify the relationship between the values of the side peaks of the correlation functions of pseudo-random sequences used in their formation and the peaks of the modules of their two-dimensional correlation function. This approach allows, without a detailed calculation of the two-dimensional correlation functions themselves in various modes of emission of complex signals, to determine their statistical characteristics using the corresponding parameters of the distribution functions of the side peaks of the correlation functions. The two-dimensional correlation function of various complex signals was calculated and their statistical characteristics were studied.

KEYWORDS: *Two-dimensional autocorrelation functions, uncertainty function, side peaks of the autocorrelation function, M-sequences, Gold codes, statistical characteristics.*

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1 Introduction

Currently, for various terrestrial and satellite radio systems, the task of detecting complex signals under conditions of a priori uncertainty, when their parameters, including the modulation law, is unknown, is relevant [1-3, 11]. Detection of such signals can be combined with assessment (measurement) of their parameters, such as carrier frequency and delay time [4, 7]. In this case, it is necessary to calculate the two-dimensional correlation function of the received complex signal in the receiver. Without calculating this function, it is impossible to ensure the required accuracy in estimating the parameters of complex signals [11].

Knowledge of the statistical characteristics of the side peaks of a two-dimensional correlation function can significantly simplify the study of the probabilistic characteristics of joint detection and estimation of the parameters of complex signals [5, 12].

The goal of the work is to develop a methodology for calculating the two-dimensional correlation function of complex signals on the entire frequency-time plane, corresponding to the domain of their definition, taking into account the use of various types of pseudo-random sequences in the formation of complex signals, as well as to study the parameters of the distribution functions of the side peaks of the two-dimensional correlation functions at unknown frequency and time delay of complex signals, called in this work the statistical characteristics of a two-dimensional correlation function.

2 Methodology for calculating the two-dimensional correlation function

The two-dimensional normalized correlation function, which is an analogue of the uncertainty function and characterizes the influence of the time and frequency shift of the received signal relative to the expected one, is described by the formula [7]:

$$\chi(\tau, f) = \frac{1}{E}\int_{-\infty}^{\infty} \dot{s}(t)\dot{s}(t+\tau)e^{i2\pi ft}dt, \tag{1}$$

where *E* is the energy of complex signals; τ – time delay of the reference signal relative to the received one; *f* – the difference in frequencies of these signals; $\dot{s}(t)$ – complex envelope of complex signals. Its time representation *s*(*t*) is described as follows:

$$s(t) = a\sum_{j=1}^{N_э} d_j cos[2\pi f_0(t-(j-1)T_e)], \tag{2}$$

where a – amplitude of complex signals; $d_j = \pm 1$ – symbols of pseudo-random sequences used in complex signals formation [5, 7]; f_0 – its carrier frequency; T_e – the duration of its elementary impulse.

Then

$$\dot{s}(t) = a\sum_{j=1}^{N_э} d_j s_0(t-(j-1)T_e), \tag{3}$$

where

$$s_0(t) = \begin{cases} 1 & \text{if} \quad 0 \le t \le T_e \\ 0 & \text{if} \quad t > T_e \end{cases}.$$

From (1) it follows that the cross section of two-dimensional correlation function in frequency at $\tau = 0$ looks like:

$$\chi(0,f)=\frac{sin(2\pi fT_s)}{2\pi fT_s}, \tag{4}$$

where T_s – complex signals duration.

At $\tau = zT_e$ ($z=1\div N-1$ – shift of pseudo-random sequences of reference and received complex signals) from (1) it follows:

$$\chi(zT_e,f)=\frac{1}{N}\chi_э(zT_e,f)\sum_{j=1}^{N}c_{j,z}e^{-i(j-1)2\pi fT_e}, \tag{5}$$

where N – total length of pseudorandom sequences; $\chi_e(zT_e,f)$ – two-dimensional correlation function of an elementary symbol of complex signals; $c_{j,z}=d_jd_{j+z}$ – symbol of the resulting pseudo-random sequences, formed by multiplying the *j*-th and *j* + *z*-th elementary symbols of pseudo-random sequences. The formation of symbols $c_{j,z}$ is illustrated in Figure 1 $c_{j,z}$

Original complex signal	d_1	d_2	d_3	• • •	$d_{Nэ}$
×					
Shifted complex signal	d_{1+z}	d_{2+z}	d_{3+z}	• • •	$d_{Nэ+z}$
=					
$c_{j,z}$	d_1d_{1+z}	d_2d_{2+z}	d_3d_{3+z}	• • •	$d_{Nэ}d_{Nэ+z}$

Fig. 1. Formation of symbols $c_{j,z}$.

In (5) two-dimensional correlation function of an elementary symbol of complex signals:

$$\chi_e(zT_e,f)=\frac{sin(2\pi fT_e)}{2\pi fT_e}. \tag{6}$$

Then (5) is transformed by the following expression:

$$\chi(zT_e,0)=\frac{1}{N_e}\sum_{j=1}^{N_e}c_{j,z}. \tag{7}$$

Let us consider the sections set of the function $|\chi(zT_e,f)|$ at $z=1,2...N_e-1$ and we will assume that the original complex signal is repeated N_{pov} times, i.e. $N=N_{pov}N_e$. Then, taking into account (5), we obtain [6, 9]:

$$\left|\chi(zT_e, f)\right| = \frac{1}{N_{pov}N_e}\chi_e(zT_e, f)\sum_{p=1}^{N_{pov}}\sum_{j=1}^{N_e} c_{j,z}e^{-i[j-1+(p-1)N_e]2\pi fT_e}, \tag{8}$$

where $p = 1, 2, ...N_{pov}$ – repetition number of pseudorandom sequences.

The double sum in (8) is written as follows:

$$A = \sum_{l=1}^{N_{pov}N_e}\sum_{m=1}^{N_e} A_l^m, \tag{9}$$

where

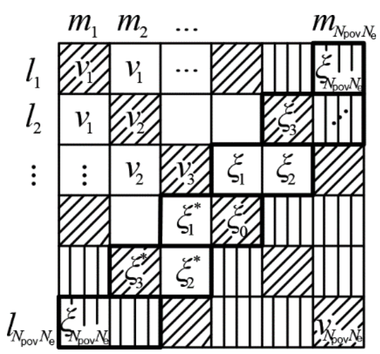
$$A_l^m = c_{j,z}e^{-i\{[(p-1)N_e+j-1]2\pi fT_e\}}, \tag{10}$$

$$l = j + (p-1)N_e, \;\; m = j + z + (p-1)N_e. \tag{11}$$

In (10), (11), firstly, the value of the parameter *z* changes, secondly, the parameter *j*, and finally *p*.

Assuming that the members of the sum A form a two-dimensional array of numbers, we define it in the form of a matrix (Table 1). In it, its elements, the modules of which have the same values, are shaded with the same inclination of the strokes (*m* – the column number of this matrix, *l* is the row number). Let us group the matrix members in Table 1 in such a way that the sum of each group is determined by the values of the peaks of correlation functions of pseudo-random sequences: aperiodic autocorrelation function, periodic autocorrelation function [2]. To do this, let's analyze the patterns in the matrix given in Table 1 [1-3].

Table 1



Let us introduce the notation: let $\xi_x(x = 0, 1, ...N_{pov}N_e - 1)$

– numbers of matrix diagonals (see Table 1), counted from its main diagonal, which has a zero number (see Table 1) (matrix elements with the designation of the diagonal number are outlined in bold lines), $\upsilon_y(y = 1, 2, ...N_{pov}N_e)$ – item number ξ_x-th diagonals [3, 8].

The results of the analysis of this matrix show that on its diagonals, symmetrical relative to the main diagonal, there are complex conjugate elements A_l^m, whose arguments are identified by their numbers ξ_x, and these arguments are equal $\xi_x 2\pi fT_e$ at the elements A_l^m, located below diagonal with the number ξ_0.

These arguments are equal $-\xi_x 2\pi fT_e$ at the elements A_l^m, located above ξ_x-th matrix diagonals. Modules of these elements A_l^m identical and equal $c_{j,z}$, where *j* and *z* are determined by numbers ξ_x and υ_y this element:

$$j = \upsilon_y - (p_y - 1)N_e \text{, where } p_y = [\upsilon_y / N_e]. \tag{12}$$

$$z = \xi_x - (p_x - 1)N_e \text{, where } p_x = [\xi_x / N_e] + 1. \tag{13}$$

[.] – integer part of a number.

From (11) and (12) it follows that on the diagonals of the matrix in Table 1 with numbers $\xi_x = (p_x - 1)N_e$, that is, multiples N_e, its elements are located A_l^m, whose modules are equal to one, i.e. $c_{j,z} = d_j^{\,2} = 1$, and do not depend on the value of the symbol of pseudo-random sequences (in Table 1 these elements A_l^m shown as ▨). Number of matrix diagonals with numbers ξ_x, multiples N_e, equals N_{pov}, and the number of A_l^m on each such diagonal it is equal $N_{pov}N_e - \xi_x$, taking into account (13) we obtain $N_e(N_{pov} - p_x + 1)$. Then

$$2\chi_{pov}(zT_e, f) = 2\sum_{p=1}^{N_{pov}-1} \frac{N_{pov} - p}{N_{pov}} cos(p2\pi fT_s), \tag{14}$$

where $\chi_{pov}(zT_e, f)$ – two-dimensional correlation function of an elementary complex signal repeated N_{pov} once, $T_s = N_e N_{pov} T_e$ – duration of pseudorandom sequences.

On the diagonals of Table 1 with numbers that are not multiples N_e, elements are found whose sum is equal to the values of the peaks of the periodic and aperiodic autocorrelation function of pseudo-random sequences corresponding to time shifts $\tau = T_e(\xi_x - (p_x - 1)N_e)$, taking into account (11), $\tau = zT_e$. Sums of matrix elements A_l^m, whose modules are equal to the values of the peaks of the periodic and aperiodic autocorrelation function of pseudo-random sequences are shown in Table 1, respectively as □ and ▥. On each of these diagonals there is $N_e(N_{pov} - (p-1))$ characters □ and $N_e - (p_x - 1)$ characters ▥ [3]. Character sums □ and ▥, which are on ξ_x-th The diagonals of the matrix in Table 1 are respectively equal to: $(N_{pov} - p)\chi_{\text{p}}(zT_e)\cos\left\{\left[\left(p-1\right)N_e + z\right]2\pi fT_e\right\}$ and $\chi_{\text{ap}}(zT_e)\cos\left\{\left[\left(p-1\right)N_e + z\right]2\pi fT_e\right\}$, where $\tau = zT_e$, $p = p_x - 1$, *z* is determined by (11), $\chi_{\text{p}}(zT_e)$ and $\chi_{\text{ap}}(zT_e)$ – values of peaks of periodic and aperiodic autocorrelation function corresponding to time shifts of pseudo-random sequences τ. Summarizing all characters □ and ▥ matrices in Table 1 and normalizing the sums with respect to $N_{pov}N_e$, we get:

$$2\chi_{\mathrm{K.p}}(zT_e,f)=2\sum_{p=1}^{N_{pov}-1}\frac{N_{pov}-p}{N_{pov}}\sum_{z=1}^{N_э-1}\chi_{\mathrm{p}}(zT_e)\times\cos\left\{\left[\left(p-1\right)N_e+z\right]2\pi fT_e\right\}. \tag{15}$$

$$2\chi_{\mathrm{K.ap}}(zT_e,f)=\frac{2}{N_{pov}}\sum_{p=1}^{N_{pov}}\sum_{z=1}^{N_e-1}\chi_{\mathrm{ap}}(zT_e)\times\cos\left\{\left[\left(p-1\right)N_e+z\right]2\pi fT_e\right\}. \tag{16}$$

Then the cross sections of a two-dimensional correlation function in a periodic mode during repetitions of the same pseudo-random sequence are described as follows:

$$\left|\chi_{\mathrm{per}}(zT_e,f)\right|=\frac{1}{N_{pov}N_{\mathrm{e}}}[1+2\chi_{pov}+2\chi_{\mathrm{K.p}}+2\chi_{\mathrm{K.ap}}]. \tag{17}$$

In aperiodic mode $N_{pov}=1$. Then from (14-17) it follows:

$$\left|\chi_{\mathrm{ap}}(zT_e,f)\right|=\frac{1}{N_{pov}N_e}[1+2\chi_{\mathrm{K.ap}}]. \tag{18}$$

$$\left|\chi_{\mathrm{K.ap}}(zT_{\mathrm{e}},f)\right|=\sum_{z=1}^{N_э-1}\chi_{\mathrm{ap}}(\tau)\cos(z2\pi fT_e). \tag{19}$$

3 Typical types of two-dimensional correlation function of complex signals

Using expressions (16, 17) describing the modules of the two-dimensional correlation function at $N_{pov}=1$, their calculation was carried out when generating complex signals based on MP and pseudo-random Gold sequences [9]. These two-dimensional correlation functions, normalized with respect to N_e, are shown in Figures 2-4, where the value is plotted along the frequency axis k ($k=fT_s\in\left[-10N_e,10N_e\right]$).

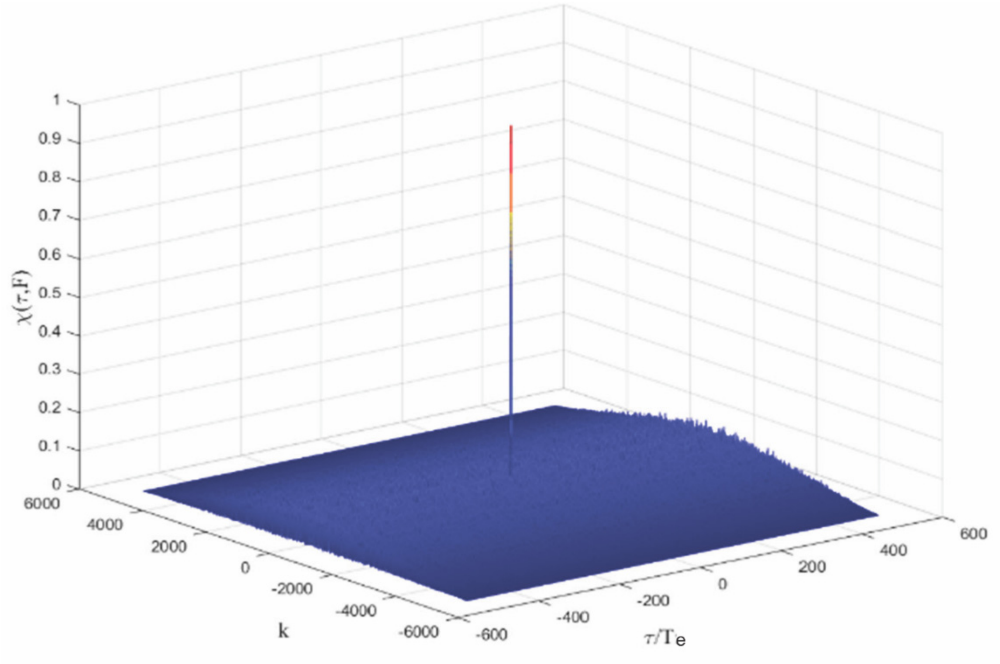


Fig. 2. Modulus of the two-dimensional correlation function of a single complex signal formed on the basis of an MF with a length N_e = 511

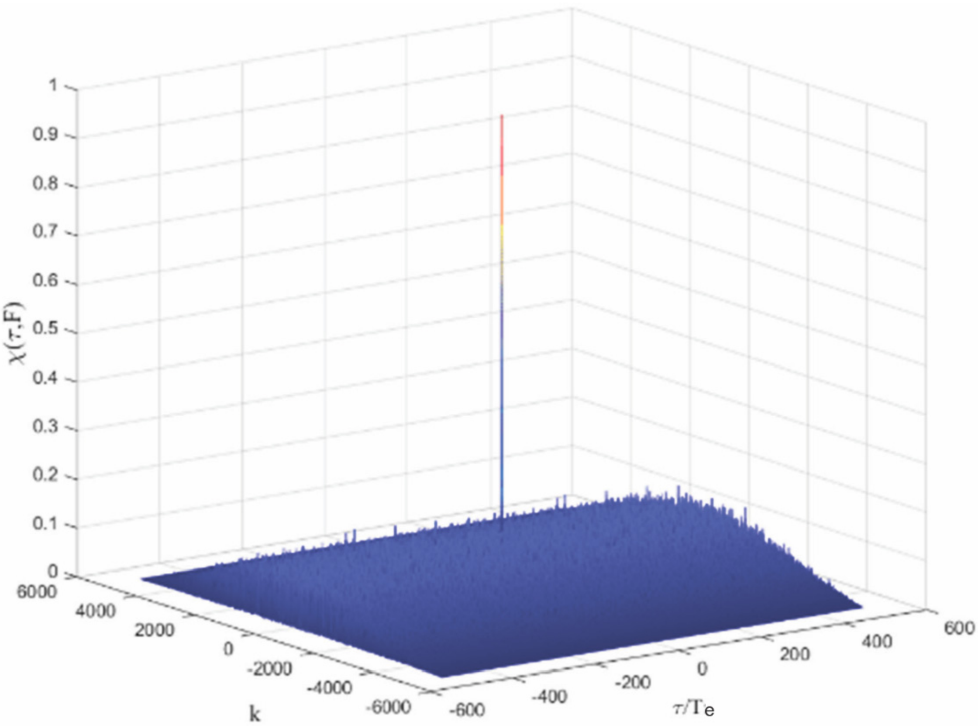


Fig. 3. The module of two-dimensional correlation function of a single complex signal generated on the basis of pseudo-random Gold sequences with a length N_e = 511

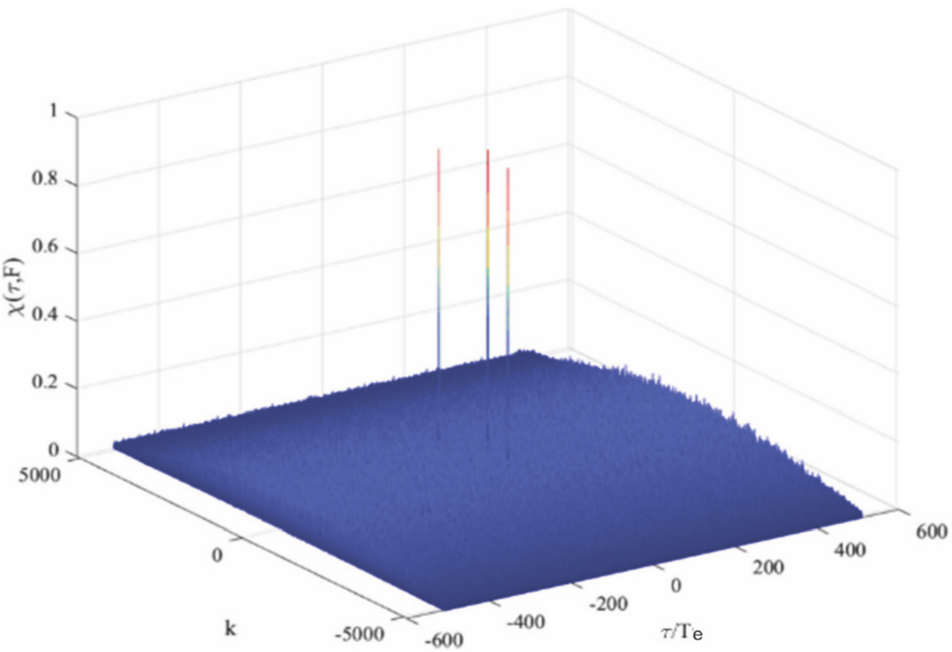


Fig. 4. The module of a two-dimensional correlation function of three copies of complex signals, shifted relative to each other in frequency and delay and formed on the basis of pseudo-random Gold sequences with a length N_e = 511

Analysis of Figures 2 and 3 shows that the two-dimensional correlation function of any complex signal has a main peak, the appearance of which does not depend on the type of pseudo-random sequences used in its formation. At the same time, the level of side peaks significantly depends on the type of such pseudo-random sequence. Analysis of Figure 4 shows that the relative shifts of the peaks of the resulting two-dimensional correlation function correspond to the shift of the three complex signals in frequency and time. The probabilistic characteristics of estimating the frequency parameters and time delay of the reference and received signals are largely determined by the level of the side peaks of the two-dimensional correlation function, compared to the level of the main peak [8, 13].

In a number of cases, people are interested in the characteristics of the real and imaginary parts of a two-dimensional correlation function. Their typical types are shown in Figures 5 and 6:

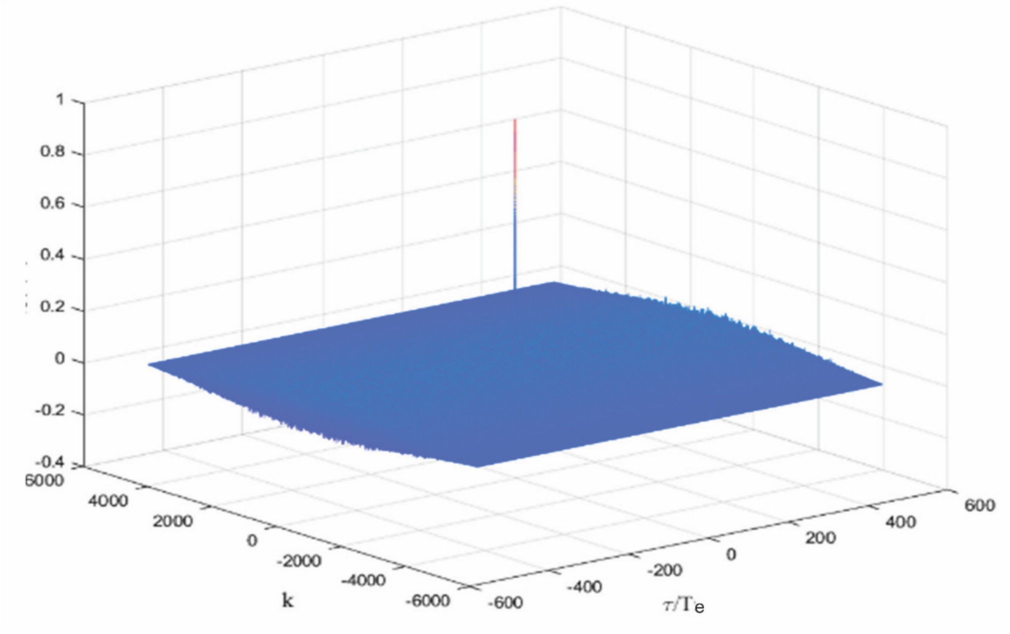


Fig. 5. The real part of the two-dimensional correlation function of a single complex signal generated on the basis of an MF with a length N_e = 511

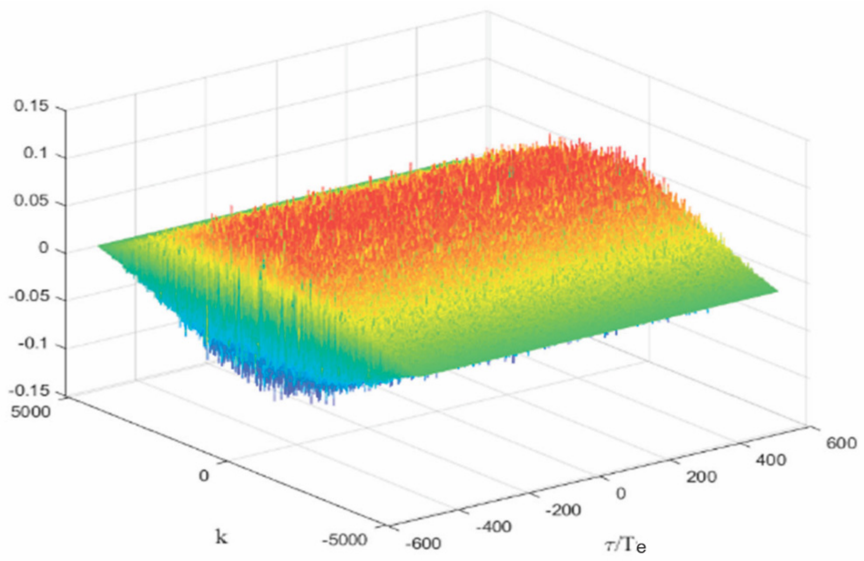


Fig. 6. Imaginary part of the two-dimensional correlation function of a single complex signal formed on the basis of an MF with a length N_e = 511

As follows from the analysis of Figures 5 and 6, the imaginary part of the two-dimensional correlation function does not have a central peak if the phase shift of the carrier frequencies of the reference and received complex signals is zero.

4 Statistical characteristics of side peaks for two-dimensional correlation function of complex signals

The following statistical characteristics [2-5] of a two-dimensional correlation function of complex signals are considered:

– mathematical expectation of the peaks of the two-dimensional correlation function of complex signals in a periodic mode:

$$m = \frac{1}{N_e N_{pov}} \chi_e(zT_e, f)(1 + 2\chi_{pov}(zT_e, f)); \qquad (20)$$

– root-mean-square deviation of the side peaks of the two-dimensional correlation function of complex signals in periodic mode in the form:

$$\sigma = (1{,}4/\sqrt{N_e}) sinc^2(\pi f T_e) \times \times \sqrt{\left\{\sigma_{\text{p}}^2 (2N_{pov}/3 - 1) + \sigma_{\text{ap}}^2 / N_{pov}\right\} / N^2{}_{pov}} - \qquad (21)$$

where σ_{p}^2 and σ_{ap}^2 – dispersion of the side peaks of periodic and aperiodic autocorrelation function.

Table 2

N_e \ $z = 1$		m			σ^2		
		Re	Im	Module	Re	Im	Module
127	MP_1	-0.005	0.003	0.0497	0.0018	0.0018	0.0011
	MP_3	-0.0049	0.0032	0.0866	0.0049	0.0049	0.0023
	MP_2	-0.0099	0.006	0.0745	0.0036	0.0036	0.0018
	(Gold's code)$_1$	0.0043	0.0037	0.0467	0.0018	0.0018	0.0014
	(Gold's code)$_3$	0.0136	0.0034	0.091	0.0055	0.0055	0.003
	(Gold's code)$_2$	0.0086	0.0058	0.0727	0.0036	0.0035	0.0019
255	MP_1	-0.0025	0.0016	0.0351	8.82E-04	8.80E-04	5.37E-04
	MP_3	0.0019	0.003	0.0674	0.0028	0.0027	9.73E-04
	(Gold's code)$_1$	-	-	-	-	-	-
	(Gold's code)$_3$	-	-	-	-	-	-
	(Gold's code)$_2$	-	-	-	-	-	-
511	MP_1	-0.0012	9.56E-04	0.0248	4.41E-04	4.40E-04	2.69E-04
	MP_3	-0.0036	0.0011	0.0461	0.0014	0.0014	7.04E-04
	(Gold's code)$_1$	0.0011	9.67E-04	0.0231	4.44E-04	4.43E-04	3.56E-04
	(Gold's code)$_3$	-0.0012	7.71E-04	0.0444	0.0014	0.0014	7.38E-04
1023	MP_1	-6.17E-04	4.38E-04	0.0176	2.20E-04	2.20E-04	1.32E-04
	MP_3	-5.47E-04	-9.16E-05	0.0305	6.70E-04	6.67E-04	4.05E-04
	(Gold's code)$_1$	5.35E-04	4.66E-04	0.0164	2.21E-04	2.20E-04	1.74E-04
	(Gold's code)$_3$	0.0017	4.70E-04	0.0299	6.70E-04	6.68E-04	4.44E-04

Continuation of Table 2

σ			Types of MP polynomials
Re	*Im*	Module	
0.042	0.0419	0.0329	M_4
0.0696	0.0697	0.0475	M_4
0.0601	0.0599	0.0423	$M_3 \oplus M_4$
0.0423	0.0421	0.0375	$M_4 \oplus M_{10}$
0.0742	0.0744	0.0545	$M_4 \oplus M_{10}$
0.0598	0.0594	0.044	$(M_4 \oplus M_{10}) \oplus (M_4 \oplus M_2)$
0.0297	0.0297	0.0232	M_2
0.0525	0.0524	0.0312	M_2
-	-	-	-
-	-	-	-
-	-	-	-
0.021	0.021	0.0164	M_1
0.0376	0.0375	0.0265	M_1
0.0211	0.021	0.0189	$M_1 \oplus M_4$
0.0368	0.0367	0.0272	$M_1 \oplus M_4$
0.0148	0.0148	0.0115	M_2
0.0259	0.0258	0.0201	M_2
0.0149	0.0148	0.0132	$M_2 \oplus M_{16}$
0.0259	0.0258	0.0211	$M_2 \oplus M_{16}$

Table 3

$N_Э$	k = 0.2	*m*			σ^2		
		Re	*Im*	Module	*Re*	*Im*	Module
127	MP_1	0.0017	-0.0047	0.0358	0.0073	6.16E-04	0.0066
	MP_3	0.0021	-0.0043	0.0596	0.0078	0.002	0.0062
	MP_2	0.0017	-0.005	0.0411	0.0078	8.90E-04	0.007
	$(\text{Gold's code})_1$	0.0183	-0.0066	0.0654	0.0105	0.0015	0.0081
	$(\text{Gold's code})_3$	0.0228	-0.0063	0.0842	0.0117	0.0026	0.0078
	$(\text{Gold's code})_2$	0.0207	-0.0082	0.0701	0.0106	0.0017	0.0079
255	MP_1	8.17E-04	-0.0023	0.023	0.0036	3.14E-04	0.0034
	MP_3	6.64E-04	-0.0022	0.0541	0.0048	0.0016	0.0034
	$(\text{Gold's code})_1$	-	-	-	-	-	-
	$(\text{Gold's code})_3$	-	-	-	-	-	-
	$(\text{Gold's code})_2$	-	-	-	-	-	-
511	MP_1	3.75E-04	-0.0011	0.0151	0.0018	1.58E-04	0.0017
	MP_3	4.59E-04	-0.0011	0.044	0.0025	0.0011	0.0017
	$(\text{Gold's code})_1$	0.004	-1.24E-04	0.0384	0.0029	7.45E-04	0.0022
	$(\text{Gold's code})_3$	0.0026	0.002	0.0541	0.0038	0.0014	0.0023
1023	MP_1	1.91E-04	-5.58E-04	0.0098	8.99E-04	7.85E-05	8.82E-04
	MP_3	2.14E-04	-5.66E-04	0.0323	0.0013	5.51E-04	8.39E-04
	$(\text{Gold's code})_1$	1.75E-04	-5.34E-04	0.0216	0.0014	3.71E-04	0.0013
	$(\text{Gold's code})_3$	1.69E-04	-5.66E-04	0.0387	0.002	8.01E-04	0.0013

Continuation of Table 3

σ			Types of MP polynomials
Re	*Im*	Module	
0.0852	0.0248	0.0814	M_4
0.0881	0.0446	0.0789	M_4
0.0884	0.0298	0.0839	$M_3 \oplus M_4$
0.1024	0.0388	0.0899	$M_4 \oplus M_{10}$
0.1081	0.0513	0.0882	$M_4 \oplus M_{10}$
0.1028	0.0417	0.0888	$(M_4 \oplus M_{10}) \oplus (M_4 \oplus M_2)$
0.0601	0.0177	0.0583	M_2
0.0689	0.0402	0.0587	M_2
-	-	-	
-	-	-	
-	-	-	
0.0424	0.0126	0.0416	M_1
0.0502	0.0327	0.0408	M_1
0.054	0.0273	0.0469	$M_1 \oplus M_4$
0.062	0.0373	0.0482	$M_1 \oplus M_4$
0.03	0.0089	0.0297	M_2
0.0364	0.0235	0.029	M_2
0.0375	0.0013	0.0362	$M_2 \oplus M_{16}$
0.0446	0.0283	0.0359	$M_2 \oplus M_{16}$

As a method for analyzing statistical characteristics, we consider the properties of the cross section of a two-dimensional function along the time axis with a frequency mismatch Δf and along the frequency axis with time mismatch $\tau = T_e$ ($z = 1$), that is, the received signal is shifted relative to the reference one by one symbol in frequency and time [10, 11].

The results of calculating the statistical characteristics of the two-dimensional correlation function for different types of complex signals are given in Tables 2 and 3, in which MP_1, MP_2, MP_3, (Gold code)$_1$, (Gold code)$_2$, (Gold code)$_3$ are the designations of MP and Gold codes with the number of repetitions 1, 2 and 3. M_1; M_2; M_3; M_4; M_{10}; M_{16} – designation of irreducible primitive polynomials, adopted in Table 3.2.1 [5].

The conducted studies show that real and imaginary parts of the two-dimensional correlation function cross sections of complex signals considered are distributed according to the Gaussian law. The Gaussian parameters change when the type of pseudo-random sequences used in the complex signals formation and mode of its radiation change. Their values are given in Table 2-3. Then the values of two-dimensional correlation function peak moduli are distributed according to Rice's law [14].

5 Conclusion

The developed methodology for calculating two-dimensional correlation function of complex signals made it possible to identify the relationship between values of the pseudo-random sequences side peaks correlation functions used in their formation and peaks of their two-dimensional correlation function modules. This approach allows, without a detailed calculation of the two-dimensional correlation functions themselves in various modes of emission of complex signals, to determine their statistical characteristics using the corresponding parameters of the distribution functions of the side peaks of the correlation functions given in [5]. However, the two-dimensional correlation function of various complex signals was calculated and their statistical characteristics were investigated.

An increase in the length of the pseudo-random sequence used to generate complex signals with an increase in the bandwidth occupied by it or an increase in its duration leads to a decrease in the level of the side peaks of the two-dimensional correlation function relative to the level of the central peak, as a result of which the shape of this function approaches the push-button one [4]. The standard deviation of two-dimensional correlation function side peaks of complex signals based on MF relative to their average value approaches zero and is significantly less than for the case of detecting three copies of same MF shifted in frequency and delay. The types of two-dimensional correlation function of complex signals generated on basis of different MPs are the same, and on the different pseudo-random Gold sequences basis – different, although they have the same values of statistical characteristics.

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