PEAK-TO-AVERAGE POWER RATIO OF SPECTRALLY EFFICIENT FDM SIGNALS

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ABSTRACT

The change of the number of subcarriers in SEFDM signal from 5 to 1024 leads to the growth of the PAPR (peak-to-average power ratio) from 4.5 to 9 dB. PAPR of SEFDM signals stays about constant (instability is no more than 5%) while normalized subcarriers frequency spacing increase from 1/2 to 1. It indicates no additional energy loss during the transition from OFDM signals to SEFDM signals, caused by an increase in the PAPR. It is proposed to use a part of the resulting gain in the frequency band to reduce PAPR. The method, based on modification of Tone Insertion method, was developed. Developed method consists in adding additional subcarriers with randomly generated manipulation symbols to the SEFDM signal. SEFDM symbol with minimal PAPR is chosen to be send. According to this method the PAPR can be reduced in average of 1.4 dB. The degree of reduction of the PAPR depends on the size of the channel alphabet of additional subcarriers (changing of the average win is 0.6 dB for BPSK to 1.4 dB for QAM-64) and the number of combinations of symbols of additional subcarriers (1.6 dB for 1000 combinations for QAM-64). According to estimation of the FPGA resources for implementation of PAPR reduction algorithm it is required relatively small resource costs compared to the formation of SEFDM signal without reduction.

KEYWORDS: *telematics, cluster, UAV, drones, optimization, information collection, base station.*

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1 Introduction

Orthogonal frequency division multiplexing (OFDM) signals are widely used in systems such as Wi-Fi, WiMAX, LTE, DVB-T2, DAB, DRM and 4G LTE downlink [1, 2]. Currently, active research is underway on signal systems for fifth generation communication networks. Multi-frequency signals with non-orthogonal multiplexing (Spectrally Efficient FDM signals) are considered as the most promising alternative to OFDM in these networks. Such signals are formed from OFDM signals by reducing the spacing between subcarriers, thereby significantly increasing the spectral efficiency of the signals [11, 12].

The main disadvantage of multi-frequency signals is the high peak factor and, as a consequence, significant underutilization of output amplifiers in terms of power [2, 5]. In portable transceiver devices, a high peak factor value (5-10 dB) causes a limitation in the signal amplitude at the transmitter output and, as a consequence, an increase in the level of out-of-band emissions [6]. The peak factor of OFDM signals has been well studied today [13-15], and a large number of different methods have been proposed to reduce it [7, 8]. However, the issue of reducing the peak factor of SEFDM signals has been poorly considered. In this work, an algorithm for reducing the peak factor is developed, based on a modification of the Tone Insertion method [9]. The developed algorithm makes it possible to reduce the peak factor by an average of 1.5 dB for the case of 103 subcarriers at moderate costs for its implementation.

The goal of the work is to reduce the peak factor of SEFDM signals.

To achieve this goal, the following tasks were solved in the work:

1. assessment of mathematical expectations and variances of the peak factor of SEFDM signals for the number of subcarriers used from 5 to 32768, various manipulation methods and compression factors;

2. construction of integral distribution functions for the peak factor of SEFDM signals;

1. analysis of existing algorithms for reducing the peak factor and proposing an option for reducing it for a large number of subcarriers;

2. assessment of the feasibility of the proposed algorithm for reducing the peak factor in programmable logic integrated circuits (FPGA).

2 Generation of SEFDM signals. Peak factor

The main characteristic of SEFDM signals is the compression factor: $\alpha = \Delta f T = \Delta f / \Delta f_{\text{орт}}$, where $\Delta f_{\text{орт}}$ – Frequency domain subcarrier spacing for orthogonal signals. For OFDM signals $\alpha = 1$. SEFDM signals occupy less bandwidth than OFDM signals, i.e. for $\Delta f < \Delta f_{\text{opt}}$ compaction factor $\alpha < 1$.

For a finite discrete signal $sN(i)$, corresponding to the nth FDM symbol, extended in duration to N samples, the following expression is satisfied:

$$s_N(i) = \frac{1}{N}\sum_{k=0}^{N-1} C_N(k) e^{j2\pi\frac{ki}{N}},\ i = 0, \ldots, (N-1),$$

where $C_{N(k)}$ – manipulation symbols (index N indicates the duration of the symbol). The signal is generated by an N-point inverse discrete Fourier transform (IDFT) from the manipulation symbols $C_{N(k)}$, after which the sequence of samples of length N obtained at the output of the IDFT is truncated to L samples [3].

The peak factor P of a SEFDM signal will be the ratio of the highest (peak) power to the average power P_{cp} of the signal $s(t)$ for the duration of one symbol T:

$$\Pi = \frac{\max\limits_{t \in [0;T]} \left\{ \left| s(t) \right|^2 \right\}}{P_{\text{cp}}}, \; P_{\text{cp}} = \frac{1}{T} \int_0^T \left| s(t) \right|^2 dt.$$

Note that the peak factor of SEFDM signal is a random variable.

3 Peak factor analysis of SEFDM signals

To analyze the peak factor of SEFDM signals, two approaches to generating SEFDM words were used: for all possible variants of manipulation symbols and for randomly generated manipulation symbols. The second approach was used to analyze the peak factor of signals with the number of subcarriers greater than 15, or the volume of the channel alphabet greater than 16.

The obtained quantitative results of the oscillation peak factor for 5 and 1024 subcarriers for two keying methods (FM-2 and QAM-64) and compression factors α: 1/2, 3/4, 7/8, 15/16, 1 are presented in Table 1. Columns $E_{\alpha,N(s)}$ show the values of the peak factor of the random SEFDM signal s averaged over the entire ensemble of implementations. Here $E_{\alpha,N(s)}$ is the mathematical expectation of signal s, depending on both α and the number of subcarriers N, the manipulation method and the number of implementations. Values for 1024 subcarriers are given for 106 implementations.

Table 1

Quantitative results of peak factor changes

FM-2	0,50	2,62
	0,75	2,70
	0,88	2,70
	0,94	2,67
	1,00	2,63
QAM-64	0,50	2,38
	0,75	2,73
	0,88	2,87
	0,94	2,87
	1,00	2,87

From Table 1 it can be seen that with an increase in the number of subcarriers, there is an increase of approximately 2.5 times (from 4.5 to 9 dB) in the peak factor values averaged over the ensemble of SEFDM signals, which practically do not change when moving from one value of the compression factor to another.

By calculating the peak factor of the corresponding SEFDM symbol for each information word and sorting the obtained results in ascending order and representing them as a column vector, it is possible to obtain the dependence of the peak factor of the SEFDM signal on the serial number in the column vector. The resulting distribution for the case α = 3/4 is presented in Figure 1.

Figure 2 shows the integral distribution functions for four manipulation methods. On it, the average values of the peak factor over the ensemble of SEFDM signals are plotted along the abscissa axis, and the values of the integral distribution function of the peak factor F(П) are plotted along the ordinate axis. From Figure 2 it can be seen that the integral distribution functions for QAM-16 and QAM-64 practically coincide, which is due to the large number of combinations of manipulation symbols.

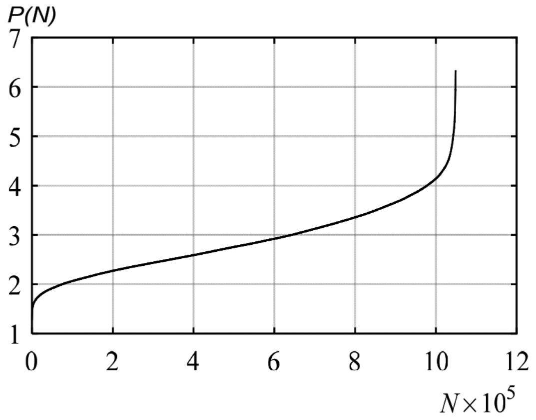

Fig. 1. Dependence of the peak factor of a SEFDM signal on the serial number in the vector-column vector for a signal with QAM-16, 5 subcarriers and α = 3/4

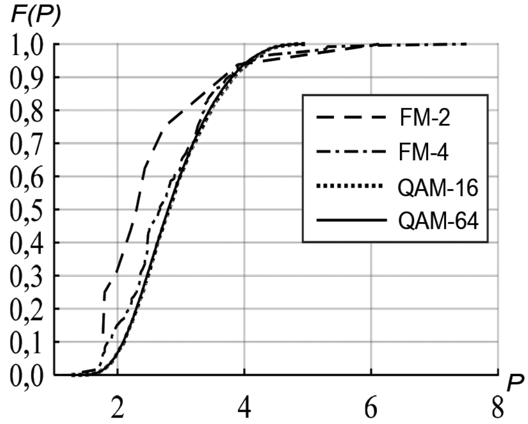

Fig. 2. Cumulative distribution functions of the SEFDM signal peak factor

4 Peak factor reduction algorithm

To reduce the peak factor of SEFDM signals, an algorithm based on the Tone Insertion method was developed [9]. The essence of the algorithm is as follows: additional subcarriers are added to the original signal (SEFDM symbol), the complex amplitudes of which are selected in such a way as to reduce the peak factor of the original signal. The developed algorithm alternates information subcarriers with additional subcarriers. In this case, the occupied frequency band becomes wider depending on the number of additional subcarriers.

The Tone Insertion method is presented in Figure 3a, its proposed modification is shown in Figure 3b. The abscissa axis shows the frequency domain, the ordinate axis shows a schematic representation of frequency subcarriers (dashed lines show additional subcarriers).

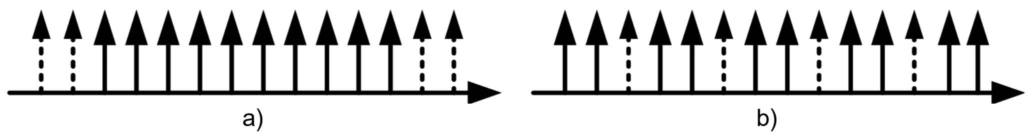

Fig. 3. Methods for adding additional subcarriers: a) along the edges of the used subcarriers (Tone Insertion); b) alternating used and additional subcarriers

Both methods, with similar parameters, provide the same reduction in peak factor on average. However, when constructing a distribution histogram and its bicubic approximation, one can see a wider spread in the ratio of the original peak factor of the SEFDM signal to the maximum possible reduction in the peak factor over the symbol duration for the second approach.

The specified spread of peak factor values is presented in Figure 4, in which the x-axis shows the ratio of the original peak factor of the SEFDM signal to the maximum possible reduction in the peak factor over the duration of the symbol, which we will call gain, and the ordinate axis shows the normalized number of sample elements. In this case, normalization is necessary due to the different widths of the pockets of the original distribution histogram (for the second method, the distribution histogram without normalization is approximately twice as high).

The gain is calculated by finding the ratio of the peak factor of the SEFDM signal before reduction (P_{orig}) in the absence of additional subcarriers to the peak factor of the SEFDM signal after adding additional subcarriers (P_s). To each information vector, presented as points on the complex plane, elements of an additional vector are added, the elements of which are formed randomly according to a uniform distribution. After that, SEFDM signals are generated in the time domain and the peak factor is calculated for each implementation of the additional vector. After calculating the peak factors for all implementations of the additional vector, the SEFDM symbol with the minimum peak factor is selected to be sent to the channel.

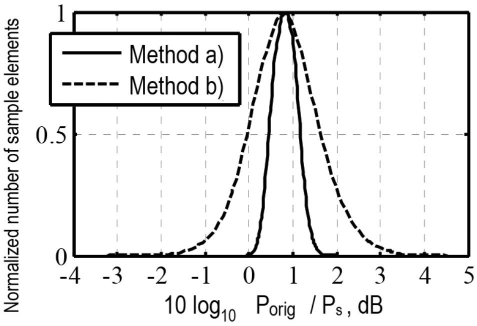

Fig. 4. Normalized bicubic approximation of the histogram of the peak factor gain distribution for 105 information vectors and 100 combinations of additional subcarriers (FM-4 for the used subcarriers, α = 3/4, FM-4 for 32 additional subcarriers)

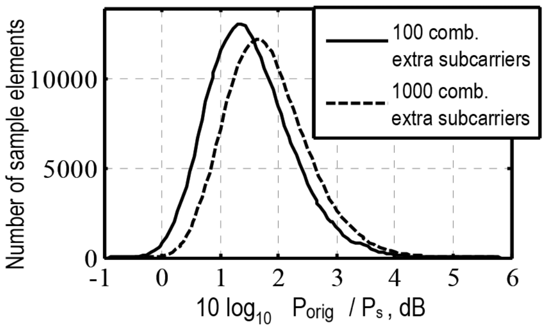

Fig. 5. Bicubic approximations of the distribution histograms of the peak factor gain depending on the number of combinations of additional subcarriers for 105 information vectors (FM-4 for the used subcarriers, α = 3/4, QAM-64 for 32 additional subcarriers)

Obviously, the more SEFDM symbols with different combinations of additional subcarriers are generated, the greater the reduction in peak factor can be obtained.

Figure 5 shows bicubic approximations of the distribution histograms of the peak factor gain depending on the number of combinations of additional subcarriers for 105 information vectors.

In the course of studying the influence of the method of manipulating additional subcarriers on the peak factor, it was found that the gain increases with increasing volume of the alphabet of the signal constellation (Fig. 6). Based on Figure 6, the change in the average gain ranges from 0.6 dB for FM-2 to 1.4 dB for QAM-64.

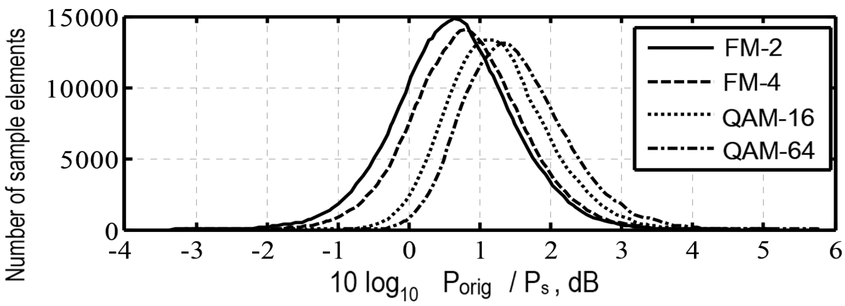

Fig. 6. Bicubic approximations of the distribution histograms of the peak factor gain depending on the method of manipulation of additional subcarriers for 105 information vectors (FM-4 for the subcarriers used, α = 3/4, 100 combinations of 32 additional subcarriers)

5 Implementation proposals

Inverse Fast Fourier transform – (IFFT) from a vector decimated by zeros with information elements at Mk positions (k = 0, ..., M, where M – 1 is the number of information elements between zero positions) is equal to the IFFT of dimension M repeated N_{fft}/M times without decimation, where N_{fft} is the dimension of the OBFT. In addition, when a zero appears at any of the M_k positions, a zero appears at the k-th position of the OBFT of dimension M. The latter property is used when adding additional subcarriers in the case of using guard intervals in the frequency domain.

The advantage of the second method over the first is the ability to use blocks (Inverse Discrete Fourier Transform – IDFT) of a smaller dimension than the main ODFT block to form SEFDM symbols from combinations of additional subcarriers. An example of interleaving for 12 information subcarriers and four additional subcarriers is presented in Figure 7a. Thus, it will be necessary to create a zero-decimated OFDM symbol once, as shown in Figure 7b, and then transition to a SEFDM symbol.

The symbols of additional subcarriers form an IFFT of dimension M (in the example in Figure 7c (M = 4, N_{fft} = 16), and the resulting vector is sequentially duplicated N_{fft}/M times, after which it is converted according to the rule for generating SEFDM symbols and added to the previously generated SEFDM symbol. This happens a predetermined number of times for various combinations of additional subcarrier keying symbols, after which the peak factor is calculated and the SEFDM symbol with its minimum value is selected.

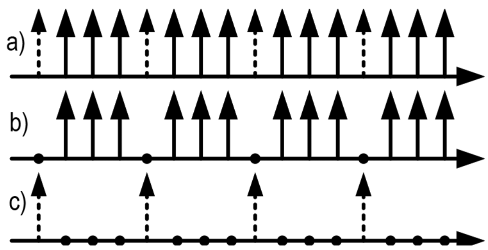

Fig. 7. Example of interleaving for 12 information subcarriers and four additional subcarriers: a) information vector in the frequency domain with additional subcarriers; b) information vector decimated by zeros; c) presentation of additional subcarriers

The proposed method for reducing the peak factor can be implemented using FPGA. To generate one reduced peak factor SEFDM symbol for a WiMAX communication system, one 1024-point IFFT and 100 32-point IFFTs are required. Considering that the latency of a 1024-point OBPF on the Xilinx board of the Virtex 7 family is 2171 clock cycles (or 8.684 μs), and the latency of a 32-point OBPF is 142 clock cycles (or 0.568 μs) [10], then in one formation of zero-decimated information vectors can be made sequentially in one 32-point block of 2171/142 $\approx$ 15 pseudo-random combinations of additional subcarriers. The number of pseudo-random combinations increases according to the increase in the number of parallel installed 32-point IFFT blocks as 15k, where k is the number of 32-point IFFT blocks.

5 Conclusion

When changing the number of subcarriers in a SEFDM signal from 5 to 1024, the average peak factor increases from 4.5 to 9 dB. The peak factor of SEFDM signals practically did not change (no more than 5%) with an increase in the frequency multiplexing factor from 1/2 to 1, which indicates the absence of additional energy loss when moving from orthogonal to non-orthogonal signals due to an increase in the peak factor.

To reduce the peak factor, it is proposed to use part of the resulting gain in the occupied frequency band. The developed modification of the Tone Insertion method, which consists in adding additional subcarriers with various combinations of keying symbols, allows reducing the peak factor by an average of 1.4 dB. The degree of reduction in the peak factor depends on the volume of the channel alphabet of additional subcarriers (the change in the average gain value ranges from 0.6 dB for FM-2 to 1.4 dB for QAM-64), as well as on the number of iterations of symbol combinations on additional subcarriers (1.6 dB for 1000 searches for QAM-64).

According to the above assessment of the used FPGA resources, implementing the peak factor reduction algorithm will require relatively small additional resource costs compared to generating a SEFDM signal without reducing it.

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