PROBABILISTIC CHARACTERISTICS OF ACCELERATED SEARCH SPREAD SPECTRUM SIGNALS

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ABSTRACT

The reasons for limiting the duration of a spread spectrum signal processed in a digital device for its accelerated search are considered, which are associated both with the instability of the frequencies of clock generators on the receiving and transmitting sides in the absence of preliminary clock synchronization, and with the conditional complexity and response speed of this device. A technique has been developed for analyzing the probabilistic characteristics of an accelerated search (detection) of a signal, which makes it possible to relate the allowable duration of its processing time and the signal-to-noise ratio at the receiver input.

KEYWORDS: *spread spectrum signals, synchronization parameters, frequency and delay uncertainty regions, search, accelerated search, probabilistic characteristics of detection*

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Introduction

Binary noise-like complex signals (NLCs), generated from binary pseudorandom sequences (PRS), are currently widely used in various digital communication systems, including satellite radio navigation systems [1, 2]. Synchronization problems are typically solved using special periodically repeating pilot signals emitted directly at the carrier frequency or at one of its quadrature components.

The main problem is the precise synchronization of NLCs in time, and less often in frequency. However, a necessary condition for the successful operation of the radio system is synchronization by the pilot signal repetition period and the clock frequency of the pseudorandom code in order to generate a reference signal at the receiving end of the NLC, synchronous in time with the received signal. This problem can be solved using a multi-stage signal search procedure, the first stage of which is performed in a special accelerated search device, where rough synchronization of the received NLC is carried out, primarily in time [3, 4].

The signal search device is a device for its combined detection and time delay parameter estimation. Therefore, the primary performance indicator is the probability of correctly detecting a signal with a certain time delay relative to a signal with a conditionally zero delay, given the false alarm probability [7, 8].

The aim of this research is to develop a method for analyzing the probabilistic characteristics of noise-like signal detection in an accelerated signal search device, taking into account the limitations on signal processing time associated with the lack of clock synchronization.

Finding sync settings

The search involves detecting a signal and measuring its parameters, typically frequency and delay time, with an accuracy corresponding to the cross-sectional dimensions of the main peak of the signal's uncertainty function (UF) [5-8]. That is, two adjacent values of any parameter can be considered indistinguishable if the difference between them is less than the cross-sectional width of the UF's central peak.

As a result, any of the synchronization parameters under consideration can be considered discrete, with the signal delay τ varying from 0 to the signal duration (repetition period), of the signal T, and ΔT from 0 to F, where is the width of the uncertainty region in frequency.

If we assume that $\Delta\tau$ and Δf_i are, respectively, the width of the interval of the delay time and frequency uncertainty region within which the values of any parameter are indistinguishable from the point of view of its evaluation, then the number of discrete values of the parameter τ is determined as $n_v = T/\Delta\tau$, and the number of values Δf of the parameter is determined as $n_{ch} = F/\Delta f_i$. But in reality, when sampling the processed signal, the values $\Delta\tau$ and Δf_i must be selected in accordance with Kotelnikov's theorem. That is, in this case, when selecting the signal sampling intervals in time and frequency in accordance with the size of the main peak of the functional class, they will be twice the signal sampling interval recommended by this theorem. Then, if the signal spectrum width is equal to ΔF_s, then $\Delta\tau \approx 1/\Delta F_s$, and $\Delta f_i \approx 1/T$.

Considering that in the case of the pseudorandom code $\Delta F_s \approx N/T$, we obtain that the total number of unknown parameters is $n_{\mathrm{v,ch}}(\mathrm{NLC}) = n_v n_{ch} = FT\Delta F_s T = FTN$, where N is the length (period) of the pseudorandom code [9].

Figure 1 shows a time-frequency plane in which the uncertainty regions of the pseudorandom code synchronization parameters – delay time and frequency – are bounded by a rectangle.

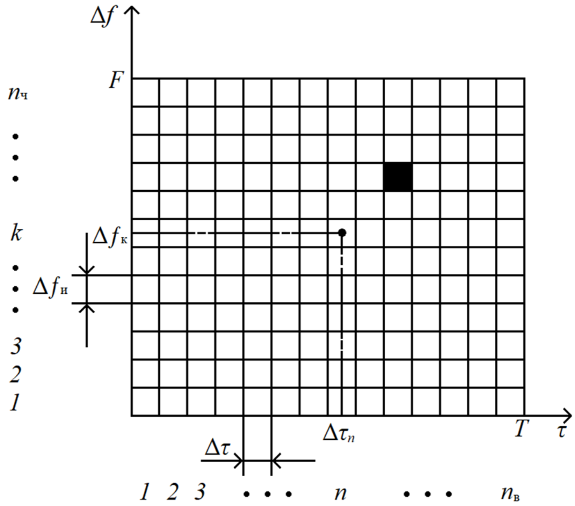

Figure 1. The uncertainty region of synchronization parameters

The parameter uncertainty region is divided by a grid into rectangular cells with sides of $\Delta\tau$ and Δf_i. The total number of cells is $n_{v,ch}$. The area of each cell is approximately equal to the area of the central peak of the FN; that is, each cell can accommodate only one of its central peaks. Therefore, the grid defines the boundaries between recognizable parameter values, and the parameters themselves correspond to the centers of the uncertainty intervals and can take any value from their total number of n_v and n_{ch}.

Thus, the discrete values of the synchronization parameters can be numbered and designated as τ_n, $n = 1,\dots,n_v$ and $\vartriangle f_k$, $k = 1,\dots,n_{ch}$.

In Figure 1, the shaded cell corresponding to the received NLS is highlighted. This is the cell that must be found during the search, after which its center must be found. However, the latter is no longer relevant to the search task.

The construction of an estimator can be based on the use of τ and Δf a set of correlators (Cor), whose reference signals are copies of the signal with discrete values of the synchronization parameters, i.e., taking into account the recommendation of Kotelnikov's theorem, $\mathrm{Re}[\dot{S}(t-\tau_n)\exp(j(2\pi(f_0+\vartriangle f_k)t+\phi))]$, $n = 1,\dots,2n_v$, $k = 1,\dots,2n_{ch}$, $\dot{S}(t)$, is the complex envelope of the received signal. The total number of correlators should be $4n_v n_{ch}$.

The block diagram of a device implementing the maximum likelihood estimate of the signal frequency and delay time, consisting of a set of correlators, is shown in Figure 2. The output signal of the correlators, after calculating the absolute values of their responses, will be the absolute value of the periodic autocorrelation function (PACF) $|\dot{\chi}(\tau,\vartriangle f)|$ (accurate to a factor corresponding to the signal energy and the additive noise component), whose values for discrete values τ_n and $\vartriangle f_k$ will appear simultaneously at the correlators' output. A decision unit (DU), which is a maximum selector, is then used. In the latter, the values of the correlator responses are compared and the maximum one is selected. The parameters of the reference signal of the correlator with the largest output response are given as the maximum plausible estimate of the frequency and delay time of the signal.

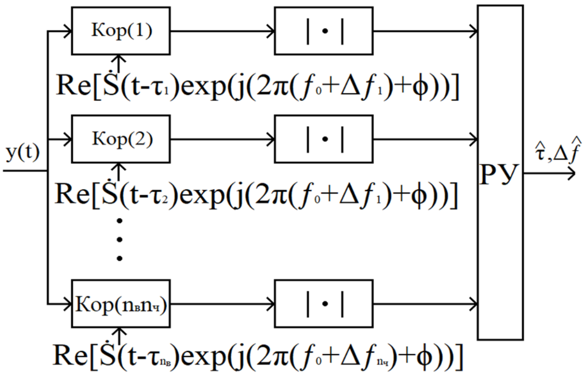

Figure 2. Device for maximum likelihood estimation of frequency and delay time based on a set of correlators

If the signal frequency is known and equal to $f_0(\Delta f = 0)$, the circuit in Figure 2 uses only 2n_v correlators to estimate the signal's time delay, but all of them can be replaced by a single matched filter (MF).

The latter is a linear device with an impulse response that is a mirror image of the useful signal, i.e., $h(t) = s(T-t)$, where *T* is the signal duration. The MF maximizes the output signal-to-noise ratio when exposed to additive white Gaussian noise, but in this context, its ability to reproduce the signal's autocorrelation function in real time is important. Indeed, the MF response is:

$$r(t) = \int_{-\infty}^{\infty} y(x)h(t-x)dx = \int_{-\infty}^{\infty} y(x)s(T-t+x)dx = \frac{1}{2}\mathrm{Re}[\int_0^T \dot{y}(x)\dot{s}(T-t+x)dx] =$$
$$= \mathrm{Re}[\{\frac{1}{2}\int_0^T \dot{Y}(x)\dot{S}(T-t+x)dx \cdot \exp(-j2\pi f_0 T)\} \cdot \exp(j2\pi f_0 t)] \cdot \quad (1)$$

As can be seen, the curly brackets in the last formula represent the complex envelope of the signal at the MF output, and:

$$|\frac{1}{2}\int_0^T \dot{Y}(x)\dot{S}(T-t+x)dx| = \frac{1}{2}|\dot{\chi}(t-T,0) + \xi_n|, \quad (2)$$

where ξ_n is the additive interference component at the MF output.

However, it is important to note that (1) assumed that the AF was matched to the input signal with an accuracy of up to the initial phase shift of its high-frequency carrier.

Thus, the AF's real-time response replicates the real part of the input signal's autocorrelation function with a factor of ½, shifted in time by the signal duration *T*.

This circumstance allows the device, to be used for the most realistic estimate of the signal's time delay.

The input signal *y*(*t*) is first processed in the SF, from whose output the signal is fed to the envelope detector (ED). In the final block, which is the decision unit (DU), the instant in time t_m when the signal at the ED output reaches its maximum value is recorded, with the estimated signal delay being equal to $\hat{\tau} = t_m - T$. The values $|\mathrm{Re}\{\dot{\chi}(t-T,0)\}|$ at the DU input appear sequentially.

In practice, a signal processor (SP) is typically a digital device whose input extracts the complex envelope of the received signal (for a signal processor generated based on a binary sequence reference sequence, this is a real function), after which it is sampled in time at a clock frequency of $f_c \approx 1/2\Delta\tau$.

Two samples are taken over the duration $T_e = T / N$ of an elementary pulse of the SP, shifted relative to each other in time by $T_e/2$. These two groups of signal samples, each taken at time intervals of duration T_e, must be processed separately – each in its own device, and a signal detection decision must be made upon its detection in either device. We will refer to them as SP1 and SP2.

It should be noted that the doubled signal sampling frequency does not exactly correspond to the clock frequency of the received signal, theoretically equal to $2f_T$, due to the instabilities of the master oscillators on both the transmitting and receiving sides. Furthermore, at the search stage, the received signal is not yet clocked. As a result, after a certain period of time, a slip will inevitably occur; that is, two SLC samples, when sampled over a time interval T_e, will fall on the same elementary pulse, or one such pulse will be missed.

Due to the shift in SLC samples at the inputs of SF1 and SF2 on T_e, a slip will never occur simultaneously at the inputs of these devices. However, the SLC processing time in these devices should not exceed the time between two consecutive slips, which can be easily estimated given the instability of the master clock frequencies on the transmitting and receiving sides.

For example, if $\frac{\Delta f_T}{f_T} = 10^{-4}$, then it is easy to calculate that the duration of the pseudorandom code that can be processed in SF1 and SF2 should not exceed approximately 5000. Therefore, it is important to be able to evaluate the efficiency of SLC search depending on the length of the PRS processed in the corresponding device.

Efficiency of Accelerated Search for SLC

The problem of searching for SLC synchronization parameters, i.e., its joint detection and parameter estimation with an accuracy corresponding to the cross-sectional dimensions of the main FC peak, is reduced to the problem of recognizing $n_{\text{v,ch}} = n_v n_{ch}$ quasi-orthogonal signals.

Indeed, each pair of parameters $\vartriangle f_k$ and τ_n corresponds to a signal $s(t, \tau_n, \vartriangle f_k)$, $n = 1, \ldots, n_v$, $k = 1, \ldots, n_{ch}$ and the FC of all these signals have non-coincident central peaks. Their mutual FCs take small values corresponding to the FC side peaks. If we assume that the FC signal shape is approximately button-shaped and the statistical characteristics of its side peaks are known, then the problem of searching for synchronization parameters is reduced to the problem of recognizing $n_{\text{v,ch}}$ quasi-orthogonal signals [9-13]. In this case $n_{\text{v,ch}} = n_v = N$.

Furthermore, a variant with a conditionally coherent signal accumulation of SLC over the duration of several repetition periods from the output of each SF was considered. In this case, the SLC can be processed in the SF over the duration of the signal repetition period, and at the input of the RA, the PACF samples are summed in parallel. The signal detection characteristics were also studied in the case where the SF can only process a portion of the NLS period.

A distinctive feature of the developed method for studying the effectiveness of the search device is the ability to construct graphs taking into account the ratio of the noise power to the signal power at the receiver input $[\frac{P_n}{P_s}]_{in}$, as well as the length of the processed pseudorandom code.

Figure 3 shows graphs of the dependence of the probabilities of correct NLS detection P_d from $[\frac{P_n}{P_s}]_{in}$ on the false alarm probability $P_{fa} = 10^{-4}$ and 10^{-6}.

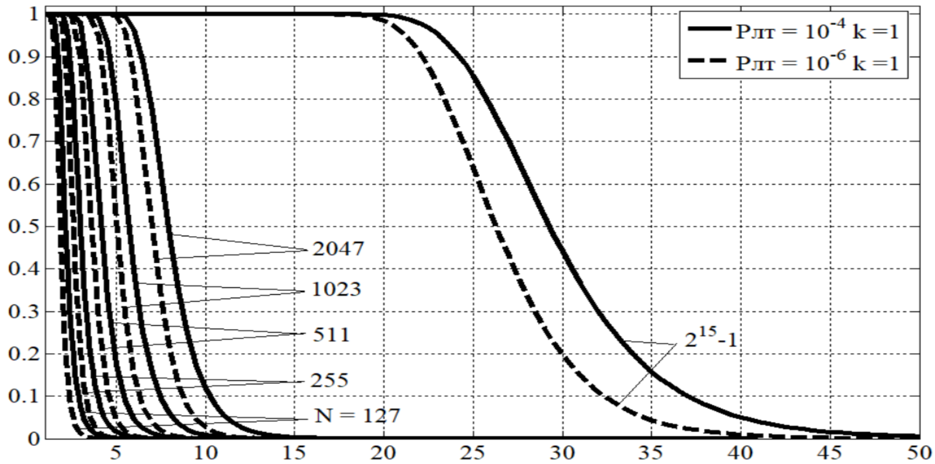

Figure 3. Dependencies on P_d from $[\frac{P_n}{P_s}]_{in}$ when forming a NLS based on MS and given $P_{fa} = 10^{-4}$ and 10^{-6} for $N = 127, 255, 511, 1023, 2047, 2^{15} - 1$

It was assumed that the signal was generated based on an M-sequence (MS) of length *N*, and the SF processes one (*k* = 1) full period of its repetition. Similar graphs, but for the case where the SF can process either a portion of the NLS period (*k* = 0.1, 0.3,…), where *k* is its portion, or a single period (*k* = 2,…, 10,…, 50) with conditionally coherent accumulation of several periods, are shown in Figures 4-9 when generating a NLS based on an MS, and in Figure 10 when generating a NLS based on a Gold code.

The developed technique allows for the analysis of the probability of error $[\frac{P_n}{P_s}]_{in}$ in the presence of several time-shifted copies of the same signal at the receiver input due to its multipath propagation.

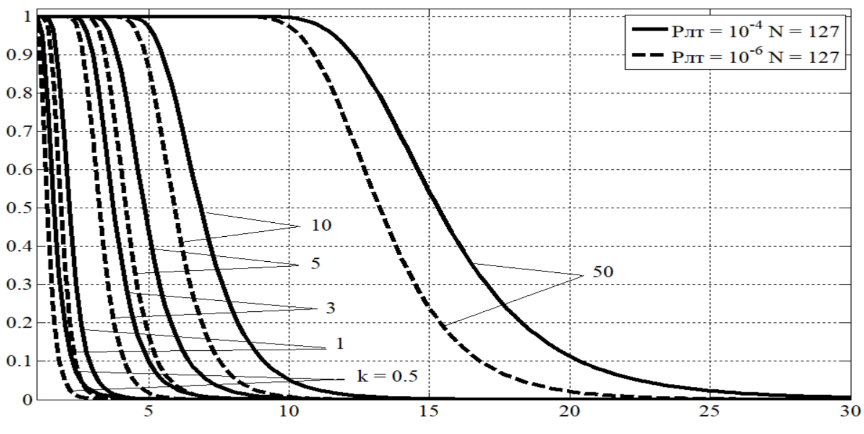

Figure 4. Dependencies on P_d from $[\frac{P_n}{P_s}]_{in}$ when forming the NLS based on MS for $N = 127$ when $k = 0.5,...,50$

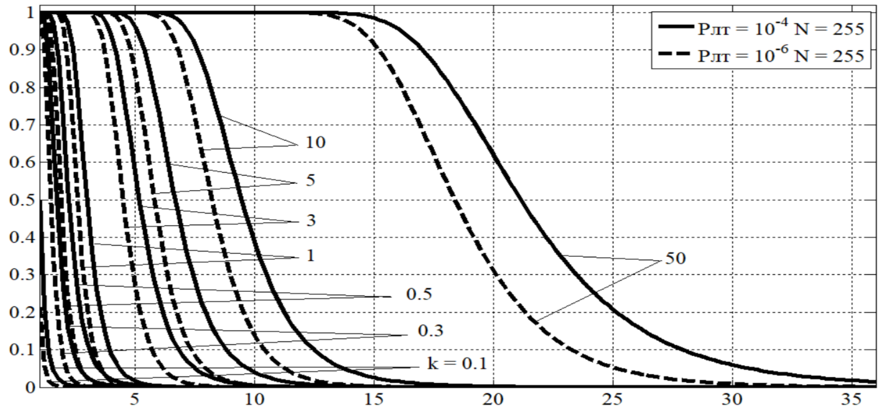

Figure 5. Dependencies on P_d from $[\frac{P_n}{P_s}]_{in}$ when forming the NLS based on MS for $N = 255$

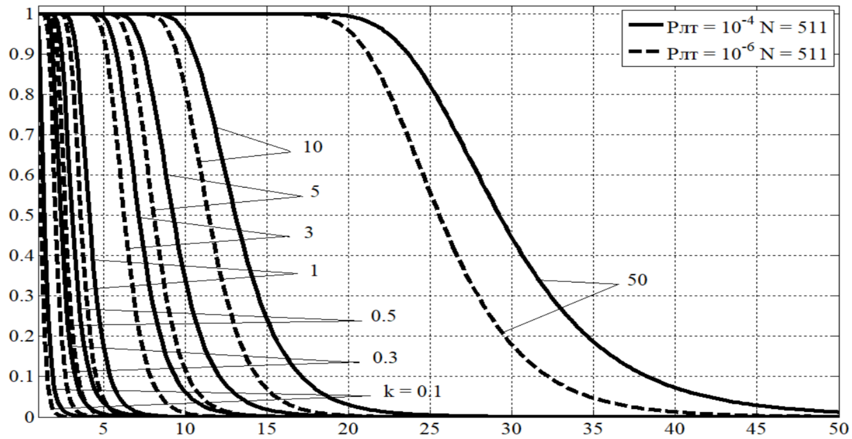

Figure 6. Dependencies on P_d from $[\frac{P_n}{P_s}]_{in}$ when forming the NLS based on MS for $N = 511$

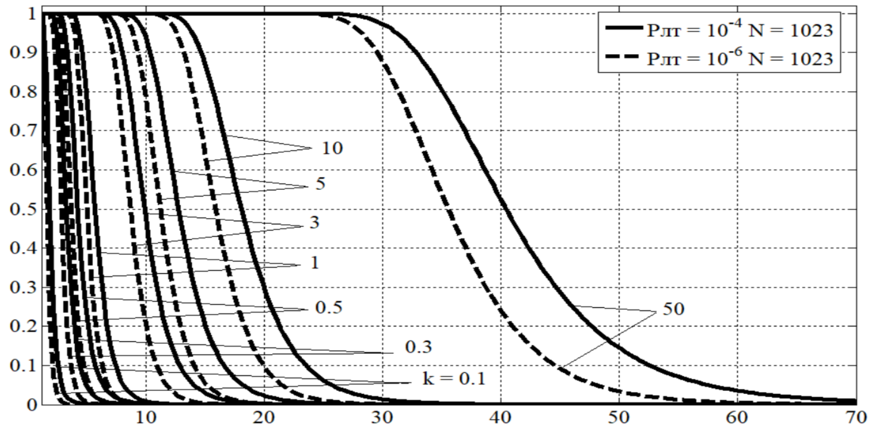

Figure 7. Dependencies on P_d from $[\frac{P_n}{P_s}]_{in}$ when forming the NLS based on MS for $N = 1023$

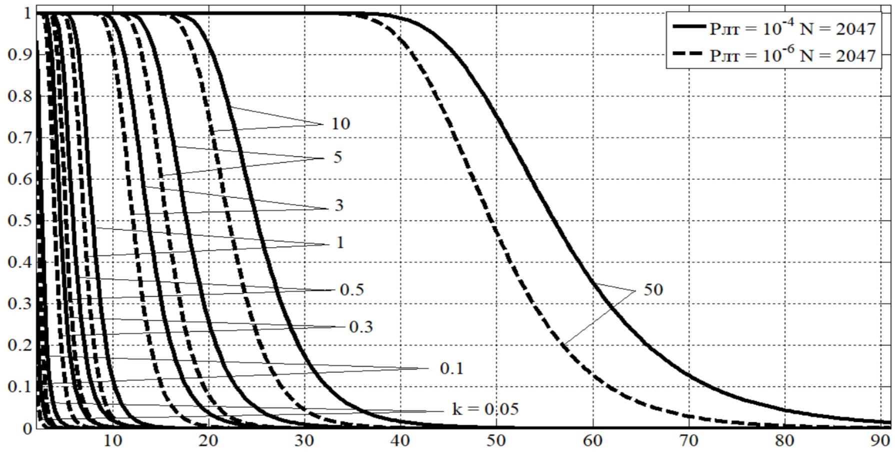

Figure 8. Dependencies on P_d from $[\frac{P_n}{P_s}]_{in}$ when forming the NLS based on MS for $N = 2047$

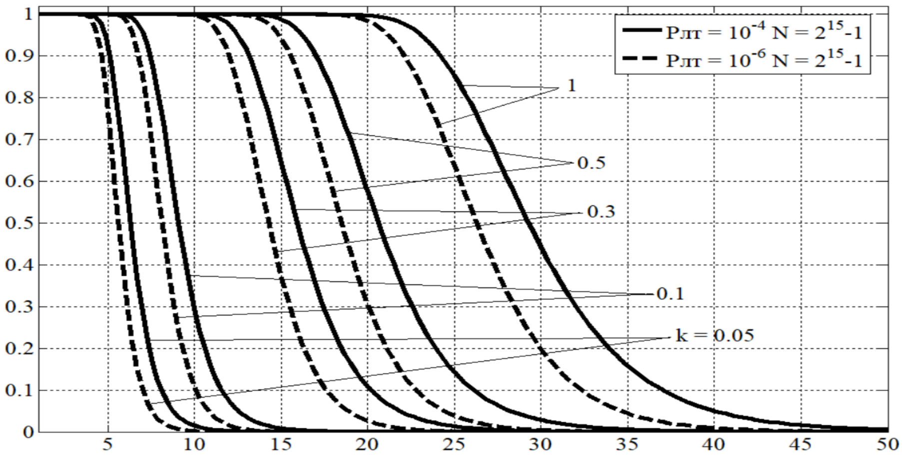

Figure 9. Dependencies on P_d from $[\frac{P_n}{P_s}]_{in}$ when forming the NLS based on MS for $N = 2^{15} - 1$

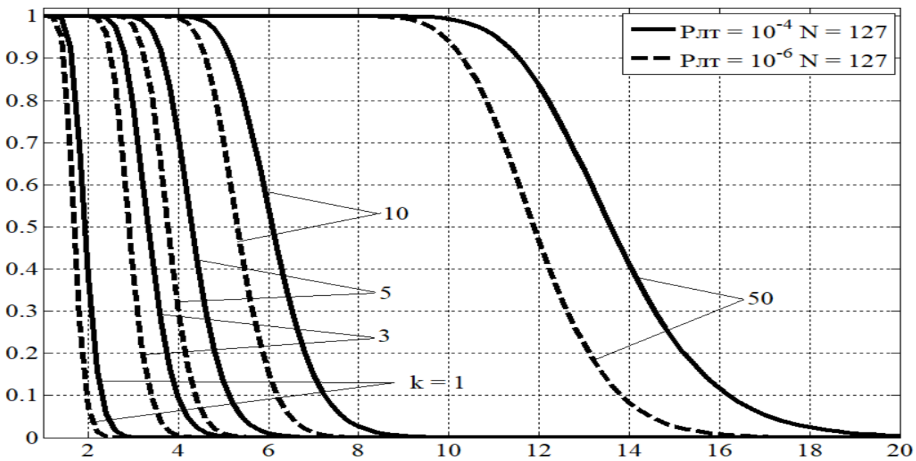

Figure 10. Dependencies on P_d from $[\frac{P_n}{P_s}]_{in}$ when forming the NLS based on the Gold code for *N* = 127

Conclusion

This article presents a developed methodology for analyzing the efficiency of a digital device for accelerated search for the time delay parameter of a complex noise-like signal, taking into account the limited processing time associated with the instability of the master clock generators on the transmitting and receiving sides.

Another factor that necessitates processing relatively short signal segments is the relative complexity of the search device and its speed. Consequently, the basic parameter for analyzing the efficiency of a noise-like signal search device is the duration of the pseudorandom code, which can be directly processed in a matched filter or correlator, as well as the permissible number of signal periods whose energy can accumulate conditionally coherently from the filter or correlator output.

Thus, the developed methodology for analyzing the efficiency of the search device allows one to evaluate its effectiveness given a priori information about the instability of the master clock generators and the noise-to-signal power ratio at the receiver input.

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