DEVICE EFFICIENCY FOR ROUGH ESTIMATION OF NOISE-LIKE SIGNAL SYNCHRONIZATION PARAMETERS

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ABSTRACT

The functional diagram of a device for roughly estimating the synchronization parameters of a periodic, noise-like, complex signal using direct spread spectrum is considered. The device is based on a unit for accelerated digital convolution of the received and reference signals. The main criterion for its effectiveness is the duration of the acquisition time for synchronization parameters with the received signal, depending on the signal-to-noise ratio at the receiver input. As shown, this duration is related to the length of the pseudorandom code used to generate the signal, the energy of which must be accumulated in the convolution unit to ensure the specified values of the probabilistic characteristics for correct estimation of the synchronization parameters with predetermined errors.

KEYWORDS: *rough estimation of synchronization parameters, synchronization of noise-like signal with direct spread spectrum, digital convolution of received and reference signals, probabilistic characteristics of synchronization parameter estimation, digital device for convolution of pseudo-random sequences*

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Introduction

A rough estimate of the synchronization parameters of a noise-like complex signal (NLS) – frequency and time delay – is usually performed with a predetermined accuracy, determined by the size of the projection of the main peak of its ambiguity function (AF) onto the frequency-time plane over which it is plotted, the shape of this peak, and the signal-to-noise ratio at the input of the receiver's decision unit (DM) [1-3]. Formally, such an estimate corresponds to the procedure for detecting and distinguishing quasi-orthogonal signals, which are distinguishable copies of the received signal, shifted relative to each other in frequency and time, against a background of white Gaussian noise [1]. The UF calculated in the receiver is formed as a multiple convolution of the received and reference NLS [4].

The aim of this work is to develop a methodology for studying the effectiveness of a rough estimate of the synchronization parameters of a periodic noise-like NLS, taking into account restrictions on the duration of the pseudorandom sequence (PRS), which can be processed in its digital convolution unit. It should be noted that periodic direct spread spectrum signals are often used in various digital radio systems as synchronization signals, as well as in satellite radio navigation systems [3, 4].

Functional device diagram for rough estimation of synchronization parameters

In accordance with the maximum likelihood criterion, a rough estimate of the frequency and time delay of the NLS, formed on the basis of a binary PRS, against the background of white Gaussian noise can be carried out in a device, the functional diagram of which is shown in Figure 1.

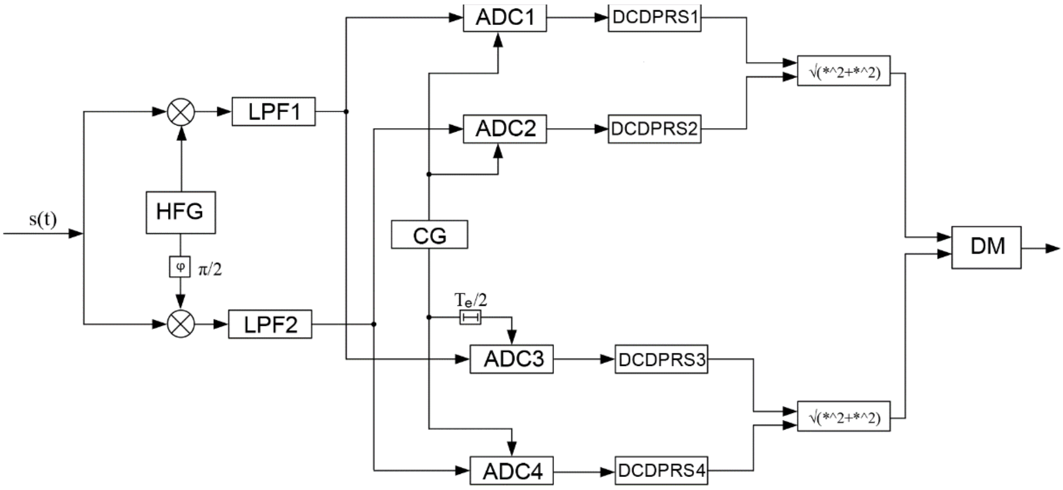

Figure 1. Functional device diagram for rough estimation of frequency and time delay parameters

Its successful operation, focused solely on estimating the time delay of the synchronization system, is possible only if the deviation of its carrier frequency from its known value is no greater than the permissible value [1, 2, 6-8]. Therefore, if the basic element of the receiving device is a digital PRS convolution device (DCDPRS), on the basis of which the synchronization system is formed, then sequential discrete tuning of the reference frequency signal at its input is necessary in order to isolate the corresponding video signal. In a more complex version of the device for rough estimation of synchronization parameters, it is necessary to parallelize the generation of reference frequencies with the same frequency step, covering the entire frequency uncertainty region of the synchronization system, or part of it. In the latter cases, it is necessary to simultaneously use DCDPRS, the number of which corresponds to the number of reference frequencies being generated. Thus, a rough estimate of the frequency of the received synchronization system will also be realized.

In Figure 1 shows the delay time estimation block of the SLC, when the high-frequency generator (HFG) generates one reference frequency f_0, i.e., for the in-phase channel – the signal $2\cos(2\pi f_0 t + \varphi)$, and for the quadrature channel – the signal $-2\sin(2\pi f_0 t + \varphi)$. Then, when an additive mixture of the useful SLC and white Gaussian noise acts at the receiver input, the signal component at the output of LPF1 *s*(*t*) will be formed as a function $Re[\dot{S}(t)\exp(j(2\pi\Delta ft+\varphi)]$, and at the output of LPF2 – as $Im[\dot{S}(t)\exp(j(2\pi\Delta ft + \varphi)]$, where $\dot{s}(t)$ is the complex envelope *s*(*t*) [2], is the Δf difference between f_0 and the carrier frequency *s*(*t*), and φ is the random phase shift between the signals of these frequencies. Further, in order to use the digital convolution device of the PRS (DCDPRS), the received signals must be discretized in time using an ADC with a clock frequency generated by the clock generator (CG) and, according to Kotelnikov's theorem, twice the clock frequency f_c of the NLS. In this case, the frequency f_c generated by the CG must be equal to the clock frequency of the NLS$1/T_e$, and its doubling is realized by secondary sampling of the signal with the same frequency, but with a shift of half the duration of the elementary symbol of the NLS, where T_e is the duration of its elementary pulse.

It is necessary to take into account that f_c, in reality, it cannot precisely match the clock frequency of the received signal due to the instabilities of the master clock generators on both the transmitting and receiving sides. Moreover, at the stage of rough estimation of synchronization parameters, clock synchronization of the received signal cannot yet be achieved. In the case of small signal-to-noise ratios, when the latter can exceed the level of the useful signal at the receiver input by hundreds of times, clock synchronization of the ADC is performed only at subsequent stages during its refinement in the automatic time adjustment device [1, 2]. As a result, after a certain period of time, so-called slippage will inevitably occur at the output of either ADC (ADC1 or ADC2), i.e., two ADC samples will fall on the same elementary pulse, or one such pulse will be missed. However, due to the shift of the NLS samples at the inputs of DCDPRS3 and DCDPRS4 relative to the samples at the inputs of DCDPRS1 and DCDPRS2 on $T_e/2$, a slip will never occur simultaneously at the inputs of these devices. Nevertheless, it is obvious that the duration of the NLS processed in any of the USPS should not be greater than the time between two consecutive slips.

Note that, given the known relative instability of the master clock generators δ, this duration is easy to calculate, since each subsequent NLS sample in the ADC will be generated not after a time interval equal to T_e, but after $T_{\text{Э}} + \delta T_{\text{Э}}$ or $T_{\text{Э}} - \delta T_{\text{Э}}$. As a result, a slip will occur δ^{-1} after NLS samples, which corresponds to the permissible PRS length that can be processed in any DCDPRS. It is assumed that the value corresponds to the maximum possible relative deviation of the frequency of any of the master clock generators from its nominal value.

At the outputs of the DCDPRS, we obtain time samples of the in-phase and quadrature components of the AF of the NLS $\dot{\chi}(\tau, \Delta f, \varphi)$, where τ is the time shift of the received NLS relative to the reference. For averaging over φ, the modulus of this function $\dot{\chi}\left|(\tau, \Delta f)\right|$ is calculated. A decision on the value τ in the DM is made when the samples at both of its inputs, or at any of them, exceed the threshold level.

Note that if exceeds the projection size Δf of the main peak of the AF on the frequency-time plane, the NLS will most likely not be detected. Therefore, it is necessary to sequentially reconfigure f_0 each time by a step corresponding to the frequency sampling interval of the NLS, which is related to the projection size of the main peak of its AF on the frequency-time plane.

Efficiency of Accelerated Search for NLS

The performance indicator for a coarse estimate of the frequency and time delay of a synchronization system is the probability of its correct detection p_{cd} in one of the intervals of the uncertainty region for these parameters, given a false alarm probability p_{fa}, depending on the signal-to-noise ratio at the input of the receiver's decision unit (DU) in the presence of an input signal. However, this ratio depends on the duration of signal energy accumulation in the receiver. Therefore, for given values, the primary performance indicator for a coarse estimate of the synchronization parameters p_{cd} and p_{fa} will be a function of the required duration of this accumulation, i.e., the duration of the synchronization system, the convolutions of which are calculated in the receiver. However, as shown above, there are limitations on the duration of the PRS processed in the DCDPRS. Therefore, to ensure the required values in the NLS synchronization parameter p_{cd} and p_{fa} coarse estimate device for any signal-to-noise ratio at the receiver input, the use of an energy accumulator at the output of the convolution unit can be considered. The probability of false detection-discrimination of orthogonal signals against a background of white Gaussian noise can be written as follows [7]:

$$p_{\text{лт}} = 1 - (1 - p_{\text{лт}0})^m \tag{1}$$

where

$$p_{\text{лт}0} = \int_{bq}^{\infty} z \exp(-z^2/2)dz = \exp(-b^2q^2/2) \approx mp_{\text{лт}0} \tag{2}$$

is the probability of false detection of a signal that is actually absent at the receiver input during incoherent reception;

$$m = 2Nm_{\text{ч}} \tag{3}$$

and in this case is proportional to the number of two-dimensional intervals of the uncertainty region of the NLS in time and frequency, the sizes of each of which correspond to the sizes of the projection of the main peak of its AF onto the frequency-time plane (*N* – the length (period) of the PRS, $m_{ch} = F/2F_s$ is the number of reference frequencies generated at the input of the DCDPRS, and are the width of the uncertainty region in frequency and the width of the spectrum of the NLS, respectively)); *d* – is the threshold level, normalized relative to the maximum value of the signal component at the output of the DCDPRS, q^2 – Is the signal-to-noise ratio in power at the input of the receiver DM.

As follows from (2),

$$bq = \sqrt{2\ln(\frac{m}{P_{\text{ЛТ}}})} \tag{4}$$

that is, the signal-to-noise ratio required to ensure a given p_{fa}, is proportional to the square root of $\ln m$.

The probability of a correct rough estimate of the synchronization parameters of the NLS [1]:

$$p_{\text{обн}} = \int_{bq}^{\infty} z \exp(-\frac{z^2 + q^2}{2}) I_0(zq)[1 - \exp(-\frac{z^2}{2})]^{m-1} dz \tag{5}$$

where $I_0(\cdot)$ is the modified zero-order Bessel function.

Note that if the function $[1 - \exp(-\frac{z^2}{2})]^{m-1}$ is approximated by a unit jump, i.e., its values are assumed to be zero for $z < z_0$ and equal to one for $z \geq z_0$, then for $q \geq z_0$, the probability of a correct rough estimate of the synchronization parameters is approximately equal to one, where $z_0 = \sqrt{2(\ln(m-1)}$. In practice, this is impossible, since in this case, *bq* should be less than *q*. However, since the threshold value *bq* is greater than z_0, the probability of a correct rough estimate of the synchronization parameters tends to one when

$$q^2 > 2\ln(\frac{m}{p_{\text{ЛТ}}}) \tag{6}$$

From (6), it follows that to increase the probability of a correct estimate of the synchronization parameters, it is necessary to increase the signal-to-noise ratio at the receiver input.

For given p_{cd} and p_{fa}, the required signal-to-noise ratio at the receiver input, i.e., q^2, is determined by the noise-to-signal ratio at its input $[\frac{P_n}{P_s}]_{in}$ and the duration of the periodic signal energy accumulation time in it, i.e., by kN, where k – Is the fraction of the PRS period or the number of its periods, on the basis of which the SRS is formed, the energy of which is accumulated in the receiver. Thus, at the output of the NLS convolution device, the maximum value of the signal-to-noise ratio will be $q^2 = kN/([\frac{P_n}{P_s}]_{in} + \sigma^2)$, where σ^2 is the variance of the side peaks of the normalized autocorrelation function (ACF) of the PRS of length kN [4]. In this case, a Gaussian approximation of the side peaks of the ACF is considered, the values σ^2 of which for typical types of PRS used to form the NLS have been studied and are given in Table 3.2.7 of [4]. Then, according to [9]

$$p_{обн} = \int_{bq}^{\infty} z \exp(-\frac{z^2+q^2}{2}) I_0(zq)dz \qquad (7)$$

Note that, for $k < 1$, the so-called aperiodic autocorrelation function (APACF) of the PRS segment is calculated in the DCDPRS, for $k = 1$, its periodic autocorrelation function (PACF), and for $k > 1$, a combination of these functions.

For the case of NLS formation based on an M-sequence (MS), the calculated dependencies P_d on $[\frac{P_n}{P_s}]_{in}$ for $p_{fa} = 10^{-4}$ and 10^{-6}, $k = 0.1, 0.3, 0.5, 1, 3, 5, 10, 50$, $m_{ch} = 10$ are shown in Figures 2 and 3. The Dolph-Chebyshev window function used in the formation of the NLS [11-14] was taken into account in the calculations.

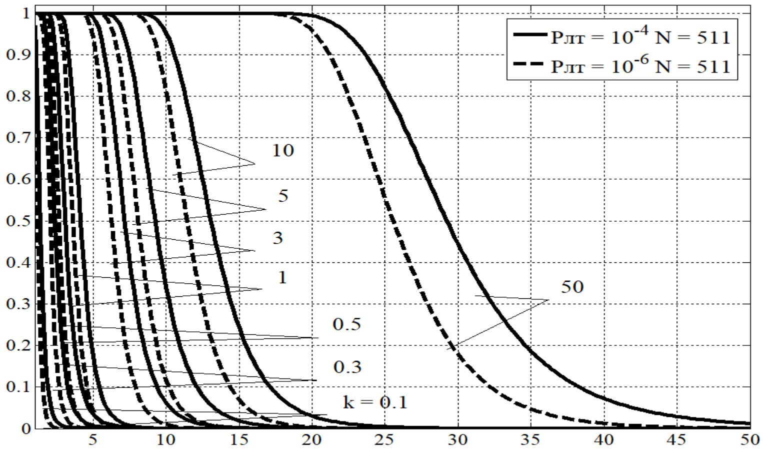

Figure 2. Probabilistic characteristics of the correct rough estimate of the synchronization parameters of the NLS based on MS at $N = 511$ (P_d on $[\frac{P_n}{P_s}]_{in}$).

As follows from the analysis of these figures, the level of the NLS ACF side peaks affects the probability of a correct rough estimate of their synchronization parameters only at low noise-to-signal ratios at the receiver input, less than 10 (i.e., when the noise power exceeds the useful signal by no more than 10 times). For weak signals, when it is necessary to accumulate the energy of several dozen NLS periods, the characteristics of the correlation functions of the used PRS can be ignored, since the noise level at the receiver input has the primary influence on the probability of a correct rough estimate of the synchronization parameters.

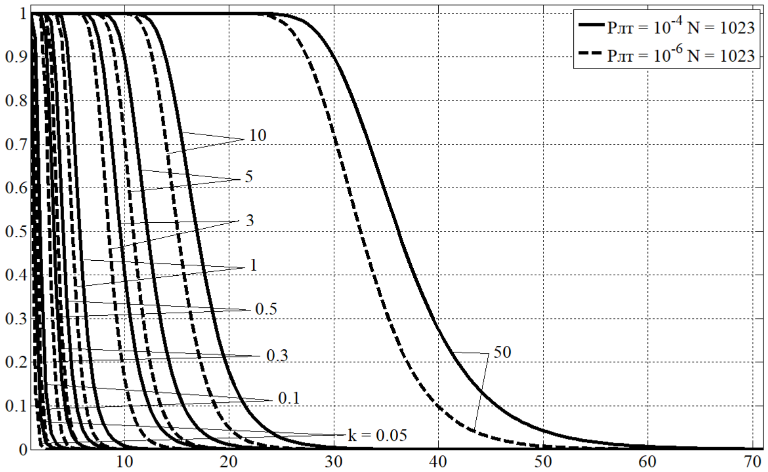

Figure 3. Probabilistic characteristics of the correct rough estimate of the synchronization parameters of the NLS based on MS at N = 1023 (P_d on $[\frac{P_n}{P_s}]_{in}$).

The considered device for rough estimation of the synchronization parameters of the NLS can also be used in the presence of several copies of the same NLS at the receiver input, shifted relative to each other in time by more than T_e, and, possibly, in frequency.

In this case, $q^2 = kN / (\left[\frac{P_{\text{ш}}}{P_{\text{с}}}\right]_{\text{вх}} + N_c\sigma^2)$, where N_c – is the number of copies of the same NLS simultaneously present at the receiver input. In addition, the probability of a correct rough estimate of the synchronization parameters is $p_{\text{d1}} = p_d^{N_c}$. The corresponding probability characteristics in the case of the simultaneous presence of three NLSs at the receiver input, generated based on MP, are shown in Figure 4, and those based on Gold codes – in Figure 5.

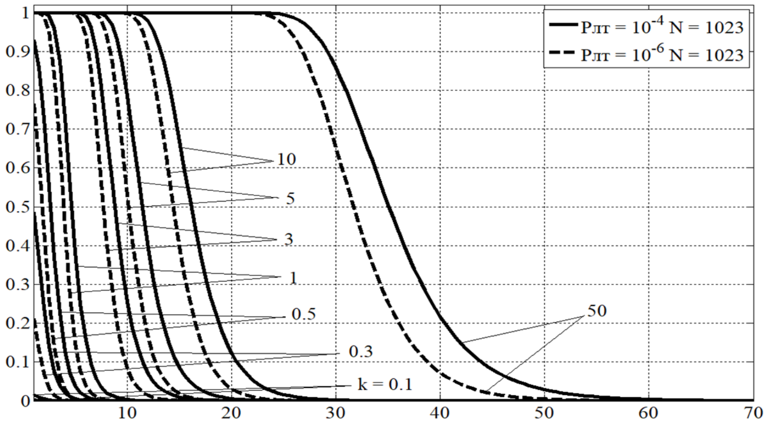

Figure 4. Probabilistic characteristics of the correct rough estimate of the synchronization parameters of three copies of the same NLS, mismatched in time and frequency, when they are formed on the basis of MS with N = 1023 (P_d on $[\frac{P_n}{P_s}]_{in}$)

As can be seen from the analysis of these figures, with increasing N_c efficiency, the estimation of the synchronization parameters of all copies of the NLS, misaligned in time and frequency, simultaneously degrades slightly, compared to the case of estimating the parameters of only one signal. Furthermore, for weak signals, that is, for large values $[\frac{P_n}{P_s}]_{in}$, it does not matter which PRS is used to generate the NLS-MS or Gold codes.

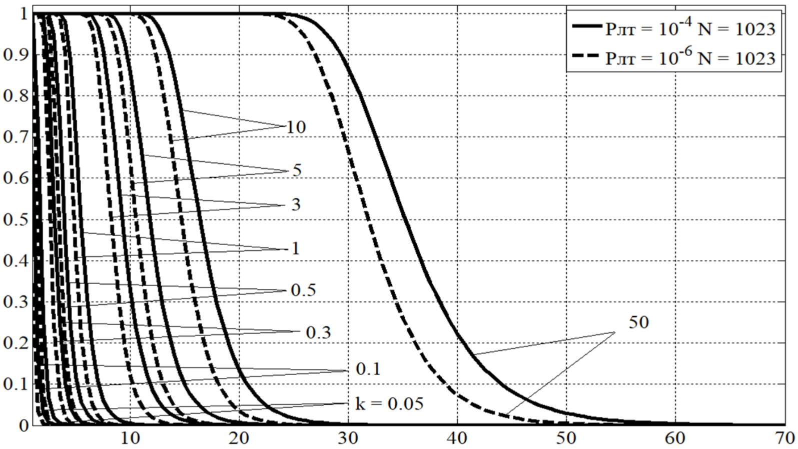

Figure 5. Probability characteristics of the correct rough estimate of the synchronization parameters of three copies of the same NLS, mismatched in time and frequency, when they are formed on the basis of the Gold code with N = 1023 (P_d on $[\frac{P_n}{P_s}]_{in}$)

In addition, it is possible to consider a variant of the DCDPRS containing blocks for calculating the convolutions of several expected NLS at once, formed on the basis of different PRS of the same type. In this case, to calculate the probabilistic characteristics of the correct estimate of the synchronization parameters in the above formulas, it is necessary to use *m* instead of mN_c, where N_c – is the number of different signals simultaneously present at the input of the receiver. In addition, $q^2 = kN/([\frac{P_n}{P_s}]_{in} + N_c(\sigma^2 + \sigma_{\text{v1}}^2))$, and the resulting probability of correct coarse synchronization is the same as in the previous case $p_{\text{d1}} = p_d^{N_c}$, where σ_{v1}^2 is the variance of the cross-correlation functions of the PRS. The values of these variances are also given in [4]. The probabilistic characteristics of the correct coarse estimate of the synchronization parameters of three NLS formed on the basis of different MS, generally mismatched in time and frequency, are shown in Figure 6, and for those based on Gold codes [10] – in Figure 7.

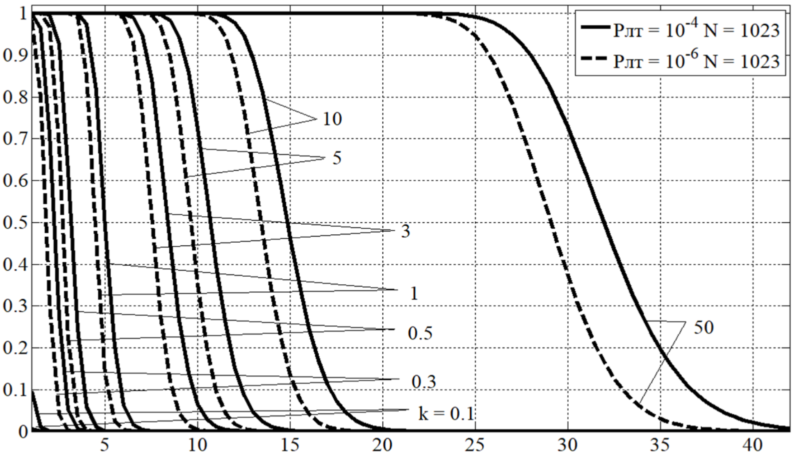

Figure 6. Probabilistic characteristics of the correct rough estimate of the synchronization parameters of three different NLS simultaneously, mismatched in time and frequency, when they are formed on the basis of MP with N = 1023 (P_d on $[\frac{P_n}{P_s}]_{in}$)

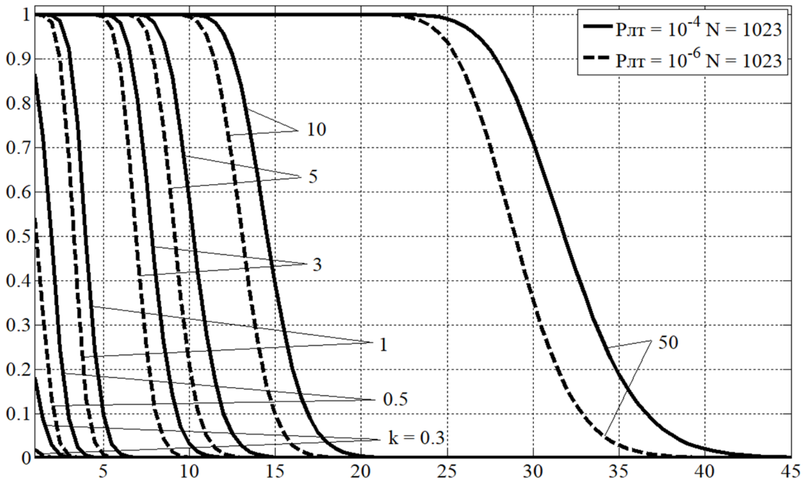

Figure 7. Probabilistic characteristics of the correct rough estimate of the synchronization parameters of three different NLS simultaneously, mismatched in time and frequency, when they are formed on the basis of Gold codes with N = 1023 (P_d on $[\frac{P_n}{P_s}]_{in}$)

As follows from an analysis of these figures, the probabilistic characteristics of a correct rough estimate of the synchronization parameters of several different NLSs simultaneously, misaligned in time and frequency, are somewhat worse than in the case of misaligned copies of the same signal. This is explained by the increased number of two-dimensional intervals of the uncertainty region that must be examined in the DM.

Rough Estimation Time for Synchronization Parameters

The time required for rough estimation of synchronization parameters depends on the time it takes to calculate the convolutions in the DCDPRS. Formally, this time is proportional to the number of blocks shown in Figure 1 and equal to the number of reference frequencies covering part of the frequency uncertainty region of the NLS. Obviously, the minimum time required to achieve synchronization in frequency and time occurs if the generated frequency grid covers the entire frequency uncertainty region at once, while the maximum occurs if the reference frequencies are generated sequentially.

If we consider only one block of the device shown in Figure 1, its operating time is determined by the speed of the digital signal processor implementing the PRS convolution procedure, the length of the processed PRS *kn*, and the convolution calculation algorithm. In the case of a simple correlation algorithm, the computational complexity of the digital convolution procedure is proportional to $(kn)^2$, but accelerated algorithms are known [5].

Conclusion

A methodology has been developed for assessing the efficiency of correctly coarsely estimating the synchronization parameters of noise-like complex signals with a known accuracy, corresponding to the size of the projection of the main peak of their uncertainty function onto the frequency-time plane over which it is plotted. This efficiency corresponds to the duration of the energy accumulation time of the signal-to-noise function required for its detection with predetermined values of their probability characteristics at any signal-to-noise ratio at the receiver input. The methodology is based on a Gaussian approximation of the side peaks of the NLS autocorrelation functions and the peaks of their cross-correlation functions.

REFERENCES

[1] L.E. Varakin, “Communication systems with noise-like signals,” Moscow: Radio and Communications, 1985. 384 p.

[2] S.F. Gorgadze, “Synchronization in infocommunication systems: a tutorial,” MTUCI. Moscow, 2022. 48 p.

[3] C. Beard, W. Stallings, “Wireless Communication Networks and Systems,” L: Pearson, 2016.

[4] V.B.v Pestryakov, V.P. Afanasyev, V.N. Gurvits, “Noise-like signals in information transmission systems,” Ed. V.B. Pestryakov. Moscow: Sov. Radio. 1973. 424 p.

[5] Vu Sy Dao, S.F. Gorgadze, “Device for accelerated search of noise-like signal,” *Technologies of the Information Society. Collection of Works of the XVI International Industry Scientific and Technical Conference.* Moscow, 2022, pp. 88-90.

[6] T.M. Gut, S.F. Gorgadze, “Characteristics of Covariance Functions and Estimation of Noise-Like Signal Parameters,” *Telecommunications and Information Technologies*. 2019. Vol. 6. No. 2, pp. 35-41.

[7] S.F. Gorgadze, “Accelerated Digital Algorithm for Synchronization of Noise-Like Signals in Time and Frequency,” *Systems for Synchronization, Generation and Processing of Signals*. 2016. Vol. 7. No. 4, pp. 16-18.

[8] S.F. Gorgadze, V.V. Boykov, “Measuring Signals with Multi-Position Subcarriers for Satellite Radio Navigation Systems,” *Radio Engineering and Electronics*. 2014. Vol. 59. No. 3. P. 264.

[9] V.I. Tikhonov, “Statistical Radio Engineering,” Moscow: Sovetskoe Radio, 1966. 219 p.

[10] V.S. Kuznetsov, I.V. Shevchenko, A.S. Volkov, A.V. Solodkov, “Generation of Gold Code Ensembles for Direct Spread Spectrum Systems,” *Proceedings of MAI*. 2017. Issue No. 96. http://trudymai.ru/

[11] V.P. Dvorkovich, A.V. Dvorkovich, “Window Functions for Harmonic Analysis of Signals,” Moscow: Tekhnosfera, 2014. 105 p.

[12] S.F. Gorgadze, Vu Sy Dao, “Detection and synchronization of weak power spread spectrum signals in a satellite radio system,” *T-Comm*, 2023, vol. 17, no.8, pp. 4-20. DOI: 10.36724/2072-8735-2023-17-8-4-20

[13] S.F. Gorgadze, Vu Sy Dao, A.V. Ermakova, “Synchronization of gold sequences based on fast transform in a truncated basis of walsh-hadamard functions,” *Radio engineering and electronics*. 2024. Vol. 69. No. 2, pp. 137-145. DOI: 10.31857/S0033849424020045

[14] S.F. Gorgadze, Vu Sy Dao, A.V. Ermakova, “Synchronization of m-sequences based on fast hadamard transform,” *Radio Engineering and Electronics*. 2024. Vol. 69. No. 2, pp. 122-136. DOI: 10.31857/S0033849424020031