COMPLEX SIGNAL DETECTION AND VECTOR-MATRIX MULTIPLICATION ALGORITHM

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ABSTRACT

The problem of joint detection and estimation of carrier frequency parameters and time delays of a low-power noise-like complex signal (NCS), its several copies mismatched in frequency and time delay, or noise-like signals of different structure, is relevant for a number of radio systems, since its solution can be used for time and frequency synchronization in information transmission channels, positioning in radio navigation systems, summation of signals during their multipath propagation or radiation by spaced repeaters, detection of all ground stations using a satellite constellation for the purpose of frequency resource monitoring, etc. The objective of the work is to improve the efficiency of digital algorithms for detecting low-power noise-like NCS, as well as to analyze the joint operation of the corresponding devices with loop circuits for tracking changes in signal parameters at a given accuracy of their final estimation in a multi-stage parallel-sequential detection and synchronization procedure, as well as to develop a unified synchronization quality criterion for a radio system. The subject of the research is digital algorithms for accelerated vector-matrix multiplication applied to the problem of detecting a set of noise-like signals; a multi-stage parallel-sequential procedure for detecting and synchronizing noise-like signals using digital synchronization devices (PSP) and analog loop circuits.

KEYWORDS: *noise-like signals, vector-matrix multiplication, Rademacher functions, Walsh-Hadamard system, complex signals*.

DOI: 10.36724/2664-066X-2026-12-2-8-25

Received: 30.01.2026
Accepted: 02.04.2026

Citation: Wu Shi Dao, "Complex signal detection and vector-matrix multiplication algorithm," *Synchroinfo Journal* **2026**, vol. 12, no. 2, pp. 8-25.

Introduction

The problem of joint detection and evaluation of the parameters of carrier frequencies and time delays of a weak-power noise-like complex signal (NLC), several of its copies mismatched in frequency and time delay, or noise-like signals of different structure, is relevant for a number of radio systems, since its solution can be used for synchronization in time and frequency in information transmission channels [1-5], positioning in radio navigation systems [6-9], summation of signals during their multipath propagation or radiation by spaced repeaters [8, 10-13], identification of all ground stations using a satellite constellation for the purpose of monitoring the frequency resource [13, 14], etc.

The detection (search) of weak noise-like signals is usually performed by long-term accumulation of their energy in the receiver [11, 6], since the signal-to-noise ratio by power at its input can be (-10 ... -40) dB, and for unknown frequencies and time delays of the received noise-like signals, its initial accumulation is usually performed using a set of correlators (Cor) or matched filters (MF) [15, 16], at the outputs of which two-dimensional correlation functions (DCF) [13, 15, 17] of the received noise-like signals or their fragments are formed. In what follows, we will consider weak noise-like signals that provide a signal-to-noise ratio by power at the receiver input in the above-mentioned range of values.

However, it should be emphasized that there is a significant limitation on the duration of the accumulation time of the noise-like signals using Cor or MF in case of the need to process the received noise-like signals with large baselines and a significant width of the region of their frequency uncertainty. The main reason for this limitation is the significant technical difficulties in the manufacture of the above-mentioned devices. As a result, the detection time of weak-power SLS can be several tens of seconds or even minutes with sequential restructuring of their detection devices by frequency [6, 10, 11].

In many cases, the Cor or SF are considered only as devices used to improve the reliability of the subsequent post-detector energy detector SLS [18, 19], in which the energy of the required number (up to several tens or even hundreds) of SLS fragments is accumulated [1, 4]. This circumstance leads to a significant decrease in the accuracy of estimating the parameters of these signals in the device for their detection, as well as the efficiency of distinguishing their mismatched copies, which is determined mainly by the size of the projection(s) of the main peak(s) of the SLS DCF onto the frequency-time plane, that is, by the characteristics of the first energy accumulation unit SLS, including a set of Cor or SF.

The subsequent energy accumulator will only ensure the accuracy of estimating the SLS parameters, corresponding to the size of this projection, with the required reliability [12, 20]. The increase in the efficiency of devices for detecting pseudorandom sequences (PRS) is associated with the development of digital algorithms for their processing [2, 3, 21, 22], which are reduced to performing the operation of discrete convolution of the pseudorandom sequence (PRS) on the basis of which it is formed, or, ultimately, the operation of vector-matrix multiplication. The limitation on the length of the PRS, the convolution of which can be performed in such a device, is associated only with the high computational complexity of the corresponding algorithm, since the problem of instability of the PRS clock generators is solved by re-sampling the input PRS with a time shift of half the duration of its elementary pulse [23], and the instability of its carrier frequency and its

Doppler shift lead only to the need for multiple repeated calculations of the discrete convolution of the PRS [14, 24].

A more accurate assessment of the parameters of the signal-to-noise ratio for the operation of a quasi-coherent receiver can be made in devices for tracking changes in these parameters in phase-locked loop (PLL) systems and automatic time control (ATC) devices [10, 20, 22, 25, 26]. That is, detection of the signal-to-noise ratio can be used to bring the devices for tracking the parameters of already detected signals to the working areas of the discriminatory characteristics of these tracking devices [20].

The scientific objective of this paper is a comprehensive examination and optimization of the procedure for jointly detecting and estimating the parameters of sets of low-power noise-like signals based on the criteria of the duration of the correct estimation of their carrier frequencies and time delays with predetermined errors and probabilities.

The problem formulated above includes a set of subproblems, the solution of which constitutes the content of this work. One of these is the justification of the choice of M-like PRS for the formation of SLSs, which are reduced to special orthogonal functions, in systems of which fast vector-matrix multiplication algorithms can be constructed based on the fast Walsh-Hadamard transform.

Previous research in this area

Significant advances in the use of fast spectral transforms in the basis of Vilenkin-Chrestenson functions and, in particular, Walsh-Hadamard in processing discrete signals were achieved in the works of V.V. Losev, V.D. Dvornikov, Y. Be'eny, K. Leung, J. Snyders, P. Li, V.M. Smolyaninov, L.E. Nazarov, and L.M. Finck [9, 7, 15, 16, 27]. In [9], group discrete multiplicative signals were proposed for the first time, their relationship with group codes was revealed, and it was shown that the optimal rule for their recognition is based on spectral analysis, the implementation of which can use fast spectral transforms. In [7, 15, 27, 28], methods for using these transforms in the theory of error-correcting coding were developed. In relation to the problem of decoding p-ary codes of maximum length, the use of fast spectral transformations in the discrete basis of Vilenkin-Chrestenson functions was considered in [12], and directly for synchronization of PRS – in [4, 9]. Also, in [4] the relationship is indicated between the problems of searching (synchronizing) SLCs during their processing in the receiver and decoding block codes constructed on the basis of cyclic shifts of their words.

For fast decoding of a code based on fast spectral transforms, it is necessary to know the method for converting its words to discrete Vilenkin-Chrestenson functions, or, when using binary codes, to Walsh functions [29]. In the case of solving the problem of code synchronization, any of its cyclic shifts must be converted to these functions [7, 9]. However, in [7 ,9, 13], the diversity of options for converting cyclic shifts of the MP to discrete Walsh functions was not identified, caused by both the diversity of multiplicative groups of the extended Galois field and the use of their cyclic shifts in such a reduction. Knowledge of such diversity makes the MP synchronization algorithm more flexible and allows us to reduce its computational complexity in certain situations, which are indicated in this dissertation. In addition, when solving the problem of synchronizing the PSP with large repetition periods, the method of identifying the correspondence between the row numbers of the Walsh-Hadamard matrix and the initial blocks of cyclic shifts of the MP, that is, in the matrix interpretation of this problem – the rows of the circulant matrix of the MP, which can be constructed in different ways, which is not considered in the works of the above-mentioned authors, becomes important.

The problem of fast synchronization of noise-like SLS, formed on the basis of Gold's PRS [8, 30-34], currently used in many radio systems, including satellite radio navigation, has not been solved. In the works of V.Yu. Mikhailov and R.B. Mazepa, devoted to this problem [33, 34], Gold's PRS, formed using the binary subclass of Gordon-Mills-Welch sequences (GMW-sequences) [7], are considered, which do not exist for N = 2^m-1, where m = 5, 7, 11, 13, 17, …. Taking into account that Gold's PRS for m = 8,12,16 are absent, it can be concluded that the method of synchronization of Gold's PRS, proposed in these works, can be applied to PRS of only four lengths, used in practical applications at present - 511,1023,16283,32567. The main problem of this approach, also used in earlier works [7, 9], but only in relation to GMV sequences, is the increase in the level of the side peaks of the normalized periodic autocorrelation functions (PACF) [4] of short PRS, to which the original longer PRS is transformed, in relation to the central peak of the normalized PACF, which is unchanged in magnitude.

Detection and discrimination of noise-like complex signals

Let the input of the receiver contain P additive copies of the same signal generated by binary phase-shift keying (PSK) of its carrier frequency; this signal over the duration of its repetition period is described as

$$s(t) = \sum_{i=0}^{N-1} d_i S_0(t - iT_e) cos(2\pi f_0 t), \quad (1)$$

where *N* is the repetition period of the binary PRS, $d_i \in \{-1,1\}$ are its elementary symbols, $i = 0,1, \ldots, (N-1)$ is the symbol number, $S_0(t)$ is the shape function of the elementary pulse of the SLS with duration T_e, f_0 is its carrier frequency. A rectangular shape of the elementary pulses of the SLS is often considered when

$$S_0(t) = \begin{cases} 1 & if \quad t \leq T_e \\ 0 & if \quad t > T_e \end{cases}.$$

In the general case, these copies differ from each other by unknown time delays, the values of the frequencies of their carrier oscillations, and the initial phases of the oscillations of these frequencies. Accordingly, for *P* = 1, the signal-to-noise ratio at the receiver input is described as $s(t - t_1 - \boldsymbol{\tau_1}, f_1 - \Delta \boldsymbol{f_1}, \Delta \boldsymbol{\varphi_1})$, where τ_1 and $\Delta \boldsymbol{f_1}$ are unknown, relatively slow shifts in the time delay and carrier frequency of a given signal-to-noise ratio relative to the constants and known values t_1 and $\Delta\varphi_1$, and Δφ_1 is a random shift in the initial phase of the signal-to-noise ratio's carrier frequency oscillation relative to the conditionally zero shift of this phase.

In accordance with the maximum likelihood criterion, the joint detection and estimation of the parameters of the SLS at *P* =1, that is, the estimation of its time shift $\widehat{\boldsymbol{\tau_1}}$, frequency $\widehat{\Delta \boldsymbol{f_1}}$ and frequency phase $\Delta\varphi_1$ relative to t_1, f_1 and zero phase shift against the background of additive white Gaussian noise corresponds to the algorithm [12]:

$$\widehat{\tau_1}, \widehat{\Delta f_1}, \widehat{\Delta\varphi_1} = \underset{\tau,\Delta f,\Delta\varphi}{\operatorname{argmax}}(Re[\dot{Z}(\tau, \Delta f, \Delta\varphi) + \xi]), \quad (2)$$

where ξ is the additive noise component at the input of the decision device (DD),

$$\dot{Z}(\tau, \Delta f, \Delta\varphi) = e^{j\,\Delta\varphi}\dot{\chi}(\tau, \Delta f), \quad (3)$$

$$\dot{\chi}(\tau, \Delta f) = \frac{1}{E_{1T_{\text{нак}}}}\int_{\mathbf{0}}^{\boldsymbol{T_n}} \dot{S}(t)\dot{S}^*(t-\tau)e^{\boldsymbol{j}2\pi\Delta f t}dt \ – \quad (4)$$

complex DCF of the SLS [7], $\dot{S}(t)$ is its complex envelope, E_{1T_n} is the value of the SLS energy accumulated over time T_n. Thus,

$$\widehat{\tau_1}, \widehat{\Delta f_1}, \widehat{\Delta\varphi_1} = \underset{\tau,\Delta f,\Delta\varphi}{\operatorname{argmax}}(Re[e^{j\Delta\varphi}\dot{\chi}(\tau, \Delta f, \Delta\varphi)]. \quad (5)$$

It is further taken into account that the estimate of Δφ is uninformative and complicates the process of estimating the frequency and time delay, as a result of which in (2) instead of the real part of the function $\dot{Z}(\tau, \Delta f, \Delta\varphi) + \xi$, its modulus is usually considered, that is, $|\dot{Z}(\tau, \Delta f, \Delta\varphi) + \xi| = |\dot{Z}(\tau, \Delta f, \Delta\varphi)| + \xi_1$, where the interference component ξ_1 is distributed according to the Rayleigh-Rice law. Then (5) can be rewritten as:

$$\widehat{\tau_1}, \widehat{\Delta f_1} = \underset{\tau_1,\Delta f_1}{\operatorname{argmax}}(|\dot{\chi}(\tau, \Delta f, \Delta\varphi)| + \xi_1). \quad (6)$$

At the outputs of the low-pass filters (LPF) of the quadrature channels of the SLS detector, the functions $Re[\dot{S}(t)e^{\boldsymbol{j}(2\pi\Delta f t+\Delta\varphi)}] = \cos\Delta\varphi Re[\dot{S}(t)e^{\boldsymbol{j}2\pi\Delta f t}]$ and $Im[\dot{S}(t)e^{\boldsymbol{j}(2\pi\Delta f t+\Delta\varphi)}] = \sin\Delta\varphi Im[\dot{S}(t)e^{\boldsymbol{j}2\pi\Delta f t}]$, are extracted, where Δf and Δφ are the values of the unknown differences between the carrier frequency of the received SLS and the reference frequency of the quadrature receiver f_1, as well as their initial phases, respectively. Then, according to (4), two convolutions of each of them with $\dot{S}^*(t)$, which in this case is a real function, are calculated separately.

The results presented below were published by the author of this dissertation in [20, 22]. The form of $|\dot{\chi}(\tau, \Delta f, \Delta\varphi)| = |\dot{\chi}(\tau, \Delta f)|$ when forming a signal-to-noise ratio based on a magnetic field with $N_e = 1023$ in the case of a rectangular shape $S_0(t)$ is shown in Figure 1. In this case, the ranges of variation of the parameters τ and Δf correspond to the width of the interval of the uncertainty region of the received signal-to-noise ratio in time T_s and frequency *F*, respectively, where T_s is the duration (repetition period) of the signal; F is the width of the uncertainty region in frequency. Within each of these intervals, there are indistinguishable values of any of the parameters from the point of view of its evaluation. The number of distinguishable discrete values of the parameter τ is determined as

$n_в = T_s / \Delta\tau$, and the number of distinguishable values of the parameter Δ*f* is determined as $n_{ch} = F / \Delta f_i$, where $\Delta\tau, \Delta f_i$ are the sampling periods of the SLS in time and frequency.

The selected sampling periods are twice as large as the periods corresponding to Kotelnikov's theorem. In this case, $\tau = zT_э$ $(z = -n_в, \dots, -1,0,1 \dots, n_в)$ is the shift of the reference signal's pseudorandom frequency relative to the pseudorandom frequency of the received signal, $\Delta f = \frac{\gamma}{T_s}$ $(\gamma = -n_ч, \dots, -1,0,1 \dots, n_ч)$ is the shift of the reference signal's carrier frequency relative to the frequency of the received signal. If the pseudorandom frequency of the received signal shifts to the left relative to the pseudorandom frequency of the reference signal, then *z* takes on negative values, otherwise it takes on positive values. Accordingly, shifts of the received signal's carrier frequency towards values lower or higher than the frequency of the reference signal are possible. The origin of the coordinate system in this figure corresponds to the coinciding values of the delay time and frequency of the received signal and the reference signal.

Figure 1 shows the discrete function $\left|\dot{\chi}\left(zT_e, \frac{\gamma}{T_s}\right)\right|$ of dimensionless z and γ, the values of which are connected by continuous lines corresponding to the form of the function $|\dot{\chi}(\tau, \Delta f)|$ between adjacent points of the discrete function corresponding to it. Also shown in this figure are the sections of the DCF along the time axis $|\dot{\chi}(zT_e, 0)|$ and along the frequency axis $\left|\dot{\chi}\left(0, \frac{\gamma}{T_s}\right)\right|$. The first zero values of $|\dot{\chi}(zT_e, 0)|$ are achieved at $z = \pm 1$, as a result of which the width of the section of the main peak of the DCF in time is $2T_e = 2/f_T$, from which it follows that the accuracy of estimating the delay time of the SL increases with an increase in the width of its spectrum, where $f_T = 1/T_e$ is the clock frequency of the SL. At the same time, as follows from (1.4), the width of the cross-section of the DCF of the frequency axis is determined by the values $\gamma = \pm 1$, at which $\left|\dot{\chi}\left(0, \frac{\gamma}{T_s}\right)\right| = 0$, therefore the width of the DCF along the frequency axis will be $2/T_s$, and $T_s = T_n = T_e N$, where *N* is the number of elementary pulses of the SLS, the energy of which is accumulated in the receiver.

Thus, with increasing N, the accuracy of the SLS frequency estimation also increases. It is generally believed that if $\Delta f_1 > 1/3T_n$, then the probability of detecting the SLS is low, since in this case the level of the useful signal at the RU input, corresponding to the DCF value, is significantly less than its maximum possible value [12, 17]. That is, for large values of Δf_1, long-term accumulation of the SLS energy in the convolution device is meaningless, but for relatively small T_n, the accumulated energy may be insufficient to detect the SLS with the required reliability. Therefore, it is necessary to reconfigure the reference frequency of the quadrature receiver with a step of $1/3T_n$ in the uncertainty region of the received SLS in frequency with a repeated calculation of the DCF.

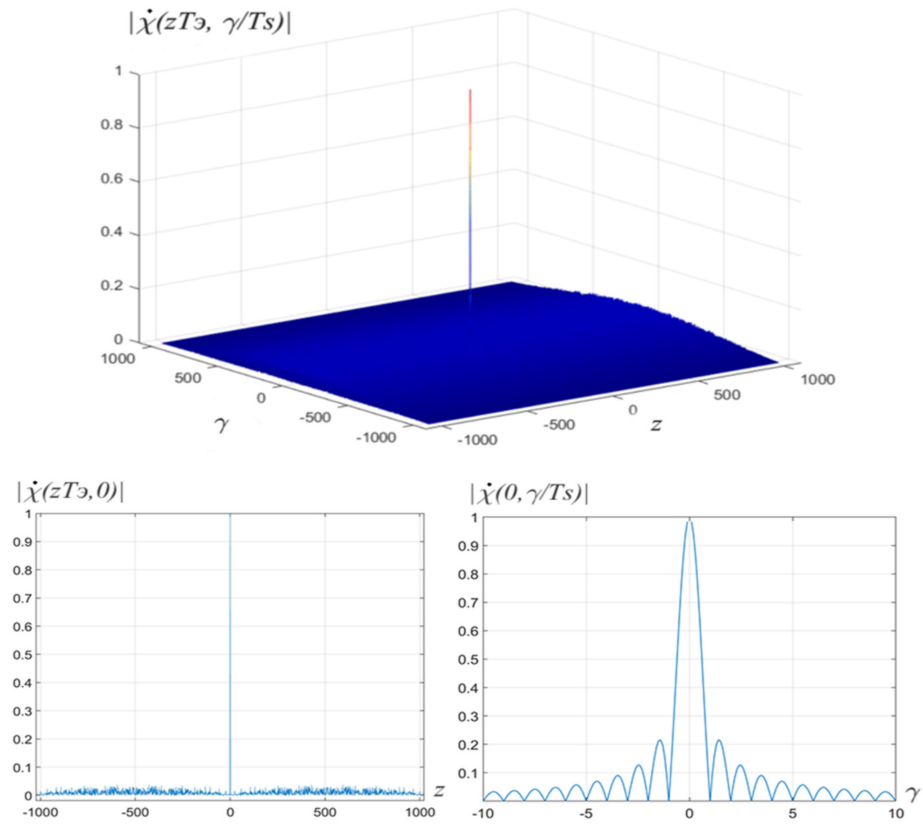

Figure 1. Typical view of the DCF module of the SLS at N_e = 1023 and its cross-sections in time and frequency.

Detection of a signal-to-noise ratio (SIR) also involves the simultaneous measurement of its frequency and delay time parameters with an accuracy corresponding to the cross-sectional dimensions of its DCF main peak. That is, two adjacent values of any parameter can be considered indistinguishable if the difference between them is less than the cross-sectional width of the central DCF peak. As a result, any of the SIR parameters under consideration can be considered discrete. When discretizing a SIR, the values of $\Delta\tau$ and Δf_i and should be selected in accordance with Kotelnikov's theorem; that is, when selecting signal sampling intervals in time and frequency in accordance with the dimensions of the main DCF peak, they will be twice the signal sampling interval recommended by this theorem. Then, if the width of the spectrum is equal to ΔF_s, and the duration of the accumulation time of its energy is equal to the period of its repetition, then $\Delta\tau \approx 1/\Delta F_s$, and $\Delta f_и \approx 1/\, T_s$. If we take into account that for $\Delta F_s \approx 1/\, T_e$, then we can estimate the total number of analyzed intervals of the uncertainty region by frequency and delay $n_{v,ch} = n_{\mathrm{v}} n_{\mathrm{ch}}$.

Figure 2 shows a time-frequency plane in which the uncertainty region of the SLS parameters – delay time and frequency – Is bounded by a rectangle.

The uncertainty region of the parameters is divided by a grid into rectangular cells with sides of $\Delta\tau$ and Δf_i The total number of cells is $n_{v,ch}$.

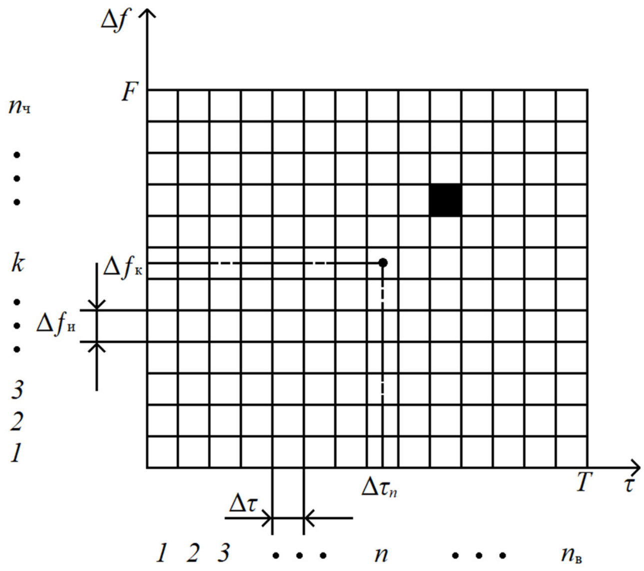

Figure 2. The uncertainty region of the parameters of the SLS.

The area of each of them is approximately equal to the area of the central peak of the DCF of the SLS, that is, each cell can accommodate only one central peak of the DCF. Therefore, the grid defines the boundaries between the recognized parameter values, and the parameters themselves can take any values from their total number n_v and n_{ch}. Thus, the discrete values of the SLS parameters can be numbered and designated as τ_n, where $n = 1, \ldots, n_v$ and Δf, where $k = 1, \ldots, n_{\mathrm{ch}}$. In Figure 3, the shaded cell corresponding to the parameter values of the received SLS is highlighted.

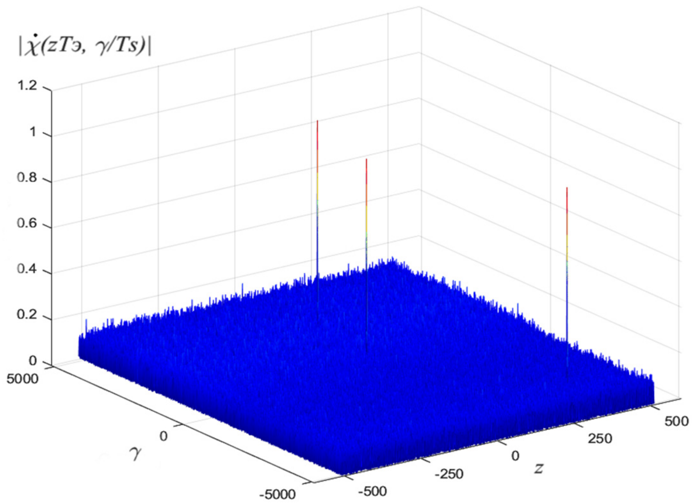

Figure 3. The DCF module of three copies of the SLS, shifted relative to each other in frequency and time delay and formed on the basis of the Gold PSP with N_e = 511.

Thus, the structural diagram of a device implementing the maximum likelihood estimate of the frequency and time delay of the SLS can be represented as a set of quadrature correlators. The output signal of the correlators after calculating the absolute values of their responses will be the absolute value of the SLS DKF $|\dot{\chi}(\tau, \Delta f)|$ (up to a factor corresponding to the signal energy and the additive noise component), whose values for discrete τ_n and Δf_k will appear simultaneously at the correlators' output. A decision unit (DU), which is a maximum selector, is then used. In the latter, the values of the correlators' responses are compared and the maximum one is selected. The reference signal parameters of the correlator with the largest output response are output as the maximum likelihood estimate of the signal frequency and time delay.

In practice, the phase-shift keyed signal generator (PSG) can be implemented as an analog device. Surface acoustic wave (SAW) devices are often used to construct phase-shift keyed signal generators (PSGs) [5], but due to limitations associated with their manufacturing technology (limited substrate sizes), they can process SAGs with a base of no more than 512 or 1024 with a sequential arrangement of two substrates.

When detecting *P* additive copies of the same signal, mismatched in frequency and time delay, the value of T_нак should be selected such that within any frequency band $[f_1 - 1/3T_n, f_1 + 1/3T_n]$, $[f_2 - 1/3T_n, f_2 + 1/3T_n]$,..., $[f_P - 1/3T_n, f_P + 1/3T_n]$ there is only one copy of the received signal with an unknown time delay, or a set of signal with the same carrier frequencies and different time delays. In this case, the nonlinear transformation in calculating $|\dot{\chi}(0, \Delta f)|$ will not lead to the appearance of significant mutual interference between copies of the same SLC, since at each moment in time only one SLC is subject to the nonlinear transformation, and mutual interference is minimized by choosing the value of T_n [1]. Figure 3 shows the form of the DCF of the group SLC for *P* = 3.

Vector-matrix multiplication in detection of noise-like signals

Digital processing of the signal in a detection device involves extracting its complex envelope and then sampling it in time at a clock rate of $f_T = 1/2T_e$. This means that for each elementary pulse of the signal, there are two samples, shifted relative to each other in time by $T_e/2$. However, the two groups of signal samples, each obtained by sampling in time at intervals of duration T_e, must be processed separately – each in its own digital device, and the decision to detect the signal should be made based on the maximum response of the two devices.

It should be noted that the doubled signal sampling frequency does not exactly correspond to the received signal's clock frequency, equal to $2f_T$, due to the instabilities of the master oscillators on both the transmitting and receiving sides. Furthermore, at the SSC detection stage, its clock synchronization has not yet been achieved. As a result, after a certain period of time, a slip will inevitably occur; that is, two SSC samples, when sampled at a time interval of T_e, will fall on the same elementary pulse, or one such pulse will be missed. However, due to the sample shift by $T_e/2$, a slip will never occur simultaneously at the inputs of two subsequent digital SSC processing devices. Nevertheless, the SSC processing time in these devices should not exceed the time between two consecutive slips, which is easily estimated from the instability of the master clock oscillators on the transmitting and receiving sides. So, if $\frac{\Delta f_T}{f_T} = 10^{-4}$, then it is easy to calculate that the duration of the PSP that can be processed in each of the digital devices should not exceed approximately 5000.

Omitting the obvious intermediate calculations and simplifications [6, 10-12], we will describe a digital algorithm for the joint detection and estimation of the parameters of a set of SLS: from the outputs of the low-pass filters (LPF) of the in-phase and quadrature channels of the receiver, analog-to-digital converters (ADC) are used in order to obtain discrete readings of the functions $Re\left[\dot{S}(t)e^{\boldsymbol{j}(2\pi\Delta ft+\Delta\varphi)}\right] = \cos\Delta\varphi Re\left[\dot{S}(t)e^{\boldsymbol{j}2\pi\Delta ft}\right]$ and $Im\left[\dot{S}(t)e^{\boldsymbol{j}(2\pi\Delta ft+\Delta\varphi)}\right] = \sin\Delta\varphi Im\left[\dot{S}(t)e^{\boldsymbol{j}2\pi\Delta ft}\right]$ at a sampling frequency of $f_T = 1/T_e$. As a result, discrete periodic signals X_1 and X_2 are formed. After re-sampling with the same clock frequency, but with a time shift of $T_e/2$, discrete signals X_3, X_4 are obtained [4]. Next, it is necessary to calculate four discrete convolutions, which in the matrix interpretation are described as: $\mathfrak{J}_{\text{ц}(1,-1)}\boldsymbol{X}_{1N}$, $\mathfrak{J}_{\text{ц}(1,-1)}\boldsymbol{X}_{2N}$, $\mathfrak{J}_{\text{ц}(1,-1)}\boldsymbol{X}_{3N}$, $\mathfrak{J}_{\text{ц}(1,-1)}\boldsymbol{X}_{4N}$, where $\mathfrak{J}_{\text{ц}(1,-1)}$ is the circulant matrix of the PRS used in the formation of the SLC of dimension N×N in the alphabet (1,-1), and $\boldsymbol{X}_{1N}$, $\boldsymbol{X}_{2N}$, $\boldsymbol{X}_{3N}$, $\boldsymbol{X}_{4N}$ are vectors that are segments of the discrete functions $\boldsymbol{X}_1$, $\boldsymbol{X}_2, \boldsymbol{X}_3$, $\boldsymbol{X}_4$ of length N, respectively. In the RU, a decision is made on the number of detected SLS and the time shifts of their PSP relative to its conditionally zero cyclic shift after calculating $\sqrt{(\mathfrak{J}_{\text{ц}(1,-1)}\boldsymbol{X}_{1N})^2 + (\mathfrak{J}_{\text{ц}(1,-1)}\boldsymbol{X}_{2N})^2}$ and $\sqrt{(\mathfrak{J}_{\text{ц}(1,-1)}\boldsymbol{X}_{3N})^2 + (\mathfrak{J}_{\text{ц}(1,-1)}\boldsymbol{X}_{4N})^2}$. The carrier frequencies of all detected SLCs will be in the frequency range $[f_1 - 1/3T_n\,, f_1 + 1/3T_n]$, $T_n = NT_e$,, where f_1 is the carrier frequency of the reference SLC used to extract the complex envelope. Then, it is necessary to change the reference frequency of the quadrature receiver by $1/3T_n$ and repeat the calculations described above. To detect all SLC copies mismatched in frequency and time delay, it is necessary to either sequentially reconfigure the reference frequency of the quadrature receiver with a step of $1/3T_n$, or to generate reference frequencies in parallel with the same step, covering the uncertainty region of the SLC in frequency during parallel calculation of convolutions.

Thus, the basis of the optimal algorithm for detecting and distinguishing SLS is the procedure of synchronization of the PRS, on the basis of which it is formed. This procedure can be described as the multiplication of the circulant matrix of the PRS $\mathfrak{J}_{\text{ц}(1,-1)}$,, the rows of which represent all of its possible cyclic shifts, by the vector X_N, obtained from the input of the receiver and containing one of the rows of this matrix, with subsequent determination of the number of the maximum component of the obtained vector. Obviously, this algorithm corresponds to the matrix form of the discrete convolution operation of the PRS, the result of which we will call its PACF [5, 10]. The order of cyclic shifts of the PRS in the rows of the matrix $\mathfrak{I}_{ц(1,-1)}$, as a rule, is not of fundamental importance [7, 9, 29]. In what follows, we will consider the MP and Gold's PRS, which are widely used at present in many radio engineering systems [6, 10, 11]. As shown below, a fast algorithm for multiplying $\mathfrak{I}_{ц(1,-1)}$ and any of its rows transposed into a column using the fast Hadamard transform (FHT) depends not only on the choice of the PSP, but also on the way this matrix is constructed.

Fast spectral transforms in the Walsh-Hadamard basis and detection of complex signals

As shown in the previous section, the algorithm for detecting a signal is reduced to digital vector-matrix multiplication, the computational complexity of which is proportional to the length of the square of the sequence of sequences on the basis of which the signal is formed, which significantly complicates its software implementation. Therefore, to generate such signals, it is advisable to use sequences of sequences that are reduced to special orthogonal functions, in systems of which fast algorithms for vector-matrix multiplication can be constructed [6, 29]. In what follows, we will consider the Walsh-Hadamard system as the initial system of orthogonal functions.

Rademacher functions and the Walsh-Hadamard system

Any discrete Rademacher system of N-th order is described by the formula:

$$r_i(x) = (-1)^{x_i} = cos\pi x_i, \quad (7)$$

where $r_i(x)$ is the Rademacher function, $x = 0,1,\dots,(N-1)$ is its integer argument, $i = 1,2,\dots,m$ is the function number in the Rademacher system, $m = log_2 N$ is the system volume, $x_i \in \{0,1\}$ are the values of the *i*-th digit of the binary representation of *x*. Thus, the Rademacher system for *m* = 4 is a 16th-order system. It consists of four functions: $r_1(x) = 11111111-1-1-1-1-1-1-1-1$, $r_2(x) = 1111-1-1-1-111\quad 11-1-1-1-1$, $r_3(x) = 11-1-111-1-111-1-111-1-1$ and $r_4(x) = 1-11-11-11--11-1$. If a number x varies from 0 to (N-1), then the value of any digit x_i of its binary representation changes periodically with a period of $2^{N/2}$. Consequently, any Rademacher function is periodic, and exactly 2^{i-1} of its periods fit on an interval of length *N*. Furthermore, it is obvious that the Rademacher functions in any system are orthogonal and odd, as a result of which they can be used to decompose only odd discrete signals, i.e., the system is not complete.

Any complete orthogonal system of Walsh functions of order *N* can be obtained from the corresponding Rademacher system of the same order. In particular, the Walsh-Hadamard system can be obtained by the rule:

$$had(h,x) = \prod_{i=1}^{n}[r_i(x)]^{h_i} = [r_1(x)]^{h_1}[r_2(x)]^{h_2} \dots [r_n(x)]^{h_n}, \quad (8)$$

where $had(h,x)$ is the Walsh function, $h = 0,\dots,(N-1)$ is its number in the Walsh-Hadamard system, and $h_1, h_2, \dots, h_n$ are the digits of the binary representation of h (h_n - is the least significant digit). Thus, the volume of the complete Walsh-Hadamard function system A_m is *N*, and the matrix A_m describing this system contains N rows. Its rows with numbers $y = 2^v, v = 0,1,2,\dots,2^{m-1}$ contain Rademacher functions, since in this case the binary representation of *y* contains a unity in only one digit.

Otherwise, the Walsh-Hadamard matrix A_m of order *N* is defined as the m-th Kronecker power of the second-order 2x2 Hadamard matrix:

$$A_m = A_2^{[m]}, \quad (9)$$

where

$$A_2^{[m]} = A_2^{(1)} \times A_2^{(2)} \times \dots \times A_2^{(i)} \times \dots \times A_2^{(m)},$$

$$A_2^{(i)} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \; i = 1,2,\dots,m,$$

$\times$ - is the notation for Kronecker matrix multiplication.

Walsh-Hadamard matrix factorization and fast Hadamard transform

The matrix A_m can be factorized, that is, represented as a simple product of matrices containing a large number of zero symbols, that is, in the form:

$$A_m = B_m^m, \quad (10)$$

where B_m is a square matrix of order $N = 2^m$. The values of its elements $b_{h,x}$ can be found using Good's theorem, which is applicable only to matrices A_m that are Kronecker powers of simpler matrices B_m, where *h* is the row number and x is the column number in which the element $b_{h,x}$ is located. According to this theorem

$$b_{h,x} = \{\lambda_{\varepsilon,r}\delta_Q^G\}, \varepsilon = h_1, r = x_m, \quad (11)$$

where $\lambda_{\varepsilon,r}$ – matrix elements A_2, $\varepsilon = 0,1$ and $r = 0,1$ – the numbers of its rows and columns, respectively,

$$\delta_Q^G = \begin{Bmatrix} 1 & \text{при } G = Q \\ 0 & \text{при } G \neq Q \end{Bmatrix} - \quad (12)$$

Kronecker delta, where $G = [\frac{x}{2}]$ is the integer part of the number $x/2$, $Q = ((h))_{N/2}$ is the remainder of dividing *h* by $N = 2^{m-1}$; h_1 is the most significant digit of the representation of the number *h* in the binary number system, where $h = h_m + 2h_{m-1} + \cdots + 2^{m-1}h_1$; $x_m$is the least significant digit of the representation of the number *x* in the binary number system.

In order to determine the structure of the matrix B_m, we note that the values of the discrete function $[x/2]$ are equal to $0,0,1,1,2,2,3,3, \ldots, 2^{m-1}-1, 2^{m-1}-1, 2^m, 2^m$, and the values of $((h))_{N/2}$ are respectively equal to $0,1,2,3, \ldots, 2^{m-1}-1, 0,1,2,3, \ldots, 2^{m-1}-1$. But then it follows from (11) that in each of its rows there will be only two elements different from zero, that is,

$$B_m = \begin{bmatrix} 1 & 1 & 0 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 1 & 1 & \ldots & 0 & 0 \\ & & & \ldots & & & \\ 0 & 0 & 0 & & \ldots & 1 & 1 \\ 1 & -1 & 0 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 1 & -1 & \ldots & 0 & 0 \\ & & & \ldots & & & \\ 0 & 0 & & & \ldots & 1 & -1 \end{bmatrix}. \tag{13}$$

Thus, the decomposition of any discrete signal, represented as a vector *X*, into the basis functions of the Walsh-Hadamard system, taking into account the factorization of the matrix A_m, is described by the formula:

$$\boldsymbol{Y} = \boldsymbol{B}_m^m \boldsymbol{X} = \boldsymbol{B}_m[\boldsymbol{B}_m \ldots [\boldsymbol{B}_m \boldsymbol{X}]]. \tag{14}$$

According to (14), column $\boldsymbol{Y_1} = \boldsymbol{B}_m \boldsymbol{X}$ is calculated first, then column $\boldsymbol{Y_2} = \boldsymbol{B}_m \boldsymbol{Y_1}$, and so on. Lastly, column $\boldsymbol{Y_m} = \boldsymbol{B}_m \boldsymbol{Y_{m-1}}$. is calculated. The procedures for calculating columns $\boldsymbol{Y_1}, \boldsymbol{Y_2}, \ldots, \boldsymbol{Y_m}$ are identical and are described by an elementary graph taking into account the Hadamard matrix factorization algorithm. An example of an elementary FPA graph for *m* = 5 is shown in Figure 4.

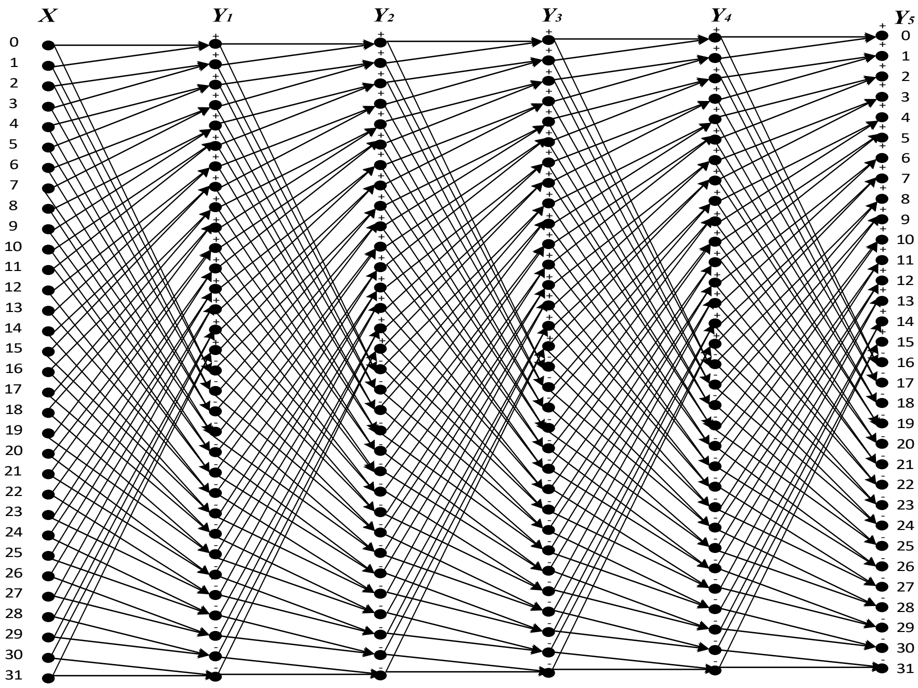

Figure 4. Example of a FPA graph with m=5.

Fast transform in the truncated Walsh-Hadamard basis

The PSPs used to form the SLS can be reduced to a limited set of rows of the Walsh-Hadamard matrix, so fast transformations in a truncated Walsh-Hadamard basis can be important. As an example, consider a fast transformation in the Rademacher system. In order to develop such an algorithm, we will take into account that the Rademacher functions are located in the rows of the Hadamard matrix $\boldsymbol{A}_m$ with numbers $y = 2^v, v = 0,1,2,\dots,2^{m-1}$ (for example, for *m* = 5 – in rows with numbers 1,2,4,8,16, for *m* = 9 – in rows with numbers 1,2,4,8,16,32,64,128,256), for *m* = 10 – in rows with numbers 1,2,4,8,16,32,64,128,256,512, etc.), as well as the possibility of representing $\boldsymbol{A}_m$ as a simple product of sparsely filled matrices $\boldsymbol{B}_m$, as described above.

The rules for fast spectral transformations in truncated Walsh-Hadamard bases are described in. By storing only *m* rows with numbers *y* in the matrix $\boldsymbol{A}_m$ out of a total of2^m, we obtain a truncated Hadamard matrix.

In the matrix $\boldsymbol{B}_1$, only m rows are stored with the same numbers $y = 2^v, v = 0,1,2,\dots,2^{m-1}$ that were stored in the matrix $\boldsymbol{A}_m$. This means that multiplying the resulting truncated matrix $\boldsymbol{B}_{1y}$ by the result of multiplying $\boldsymbol{B}_{2y} \dots \boldsymbol{B}_{my}$ by the input vector *X* requires only m summations. We obtain the matrix $\boldsymbol{B}_{2y}$ taking into account that the $(N - 2m)$ columns of the matrix B_1y consist only of zero elements. These columns must also be excluded from $\boldsymbol{B}_{1y}$, simultaneously excluding the rows of matrix $\boldsymbol{B}_2$, the numbers of which coincide with the numbers of the columns excluded in $\boldsymbol{B}_{1y}$, therefore matrix $\boldsymbol{B}_{2y}$ contains 2*m* rows, and when multiplying it by $\boldsymbol{B}_3 \dots \boldsymbol{B}_m\boldsymbol{X}$, only 2*m* summation operations will be required, in the last matrix $\boldsymbol{B}_{my}$ 2^m rows are always preserved, that is, $\boldsymbol{B}_{my} = \boldsymbol{B}$.

Thus, the number of rows stored in the matrices $\boldsymbol{B}_{sy}$, $s = 1, \dots, m$, which coincides with the number of elementary summation operations required to multiply a vector with each matrix, is described as $m + 2m + (2 * 2m - 4) + (2 * (2 * 2m - 4) - 8) + (2 * (2 * (2 * 2m - 4) - 8) - 16) + \cdots$. A recurrence formula can be derived for calculating the number of nonzero rows of the matrices $\boldsymbol{B}_{sy}$:

$$\boldsymbol{B}_s = \begin{cases} m\text{ , если } s = 1, \\ 2m\text{ , если } s = 2, \\ 2B(s-1) - 2^{s-1}\text{, если } s = 3,4,\dots, \end{cases} \qquad (15)$$

where $\boldsymbol{B}_s$ is the number of rows stored in the $\boldsymbol{B}_{sy}$ matrix. Then the number of elementary mathematical summation operations during accelerated multiplication of a matrix of Rademacher functions by a vector is:

$$S = B(1) + B(2) + \sum_{s=3}^{m} 2B(s-1) - 2^{s-1}, \qquad (16)$$

and the gain in the number of such operations, compared to simple multiplication of a matrix of the same dimension by a vector, will be $I(m) = m2^m/S$. For m = 5 it is approximately 1.84, but for typical lengths of the PBS used in constructing, for example, navigation codes, it is more significant – for *N* = 511(*m* = 9) the gain is 3 times, and for *N* = 1023(*m* = 10) – 3.4 times. Figure 5 shows the gain in the number of elementary operations for accelerated multiplication of the matrix of Rademacher functions and a vector, compared to simple multiplication.

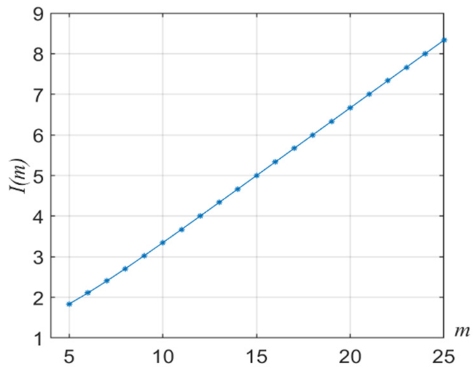

Figure 5. Gain in the number of elementary arithmetic operations during accelerated multiplication of a matrix of Rademacher functions and a vector, compared to simple multiplication.

An example of a fast transformation graph for accelerated multiplication of the Rademacher function matrix for m=5 and a vector is shown in Figure 6.

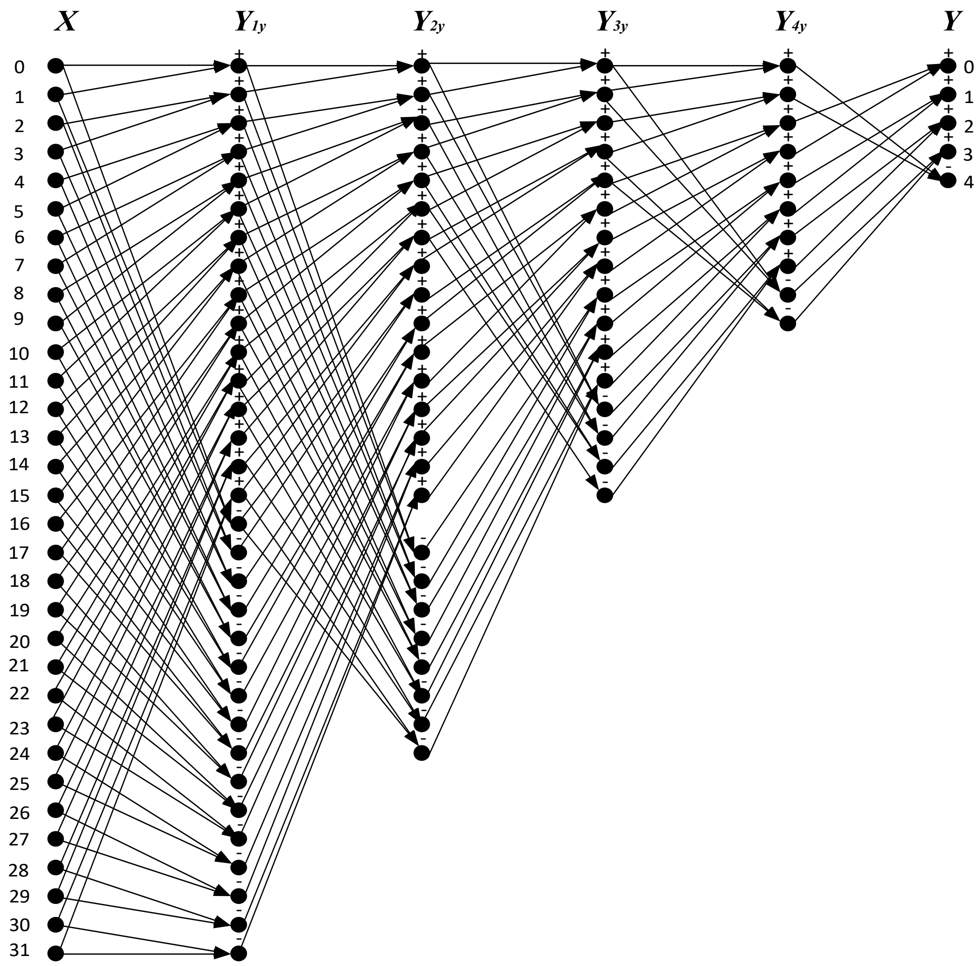

Figure 6. Fast transform graph for accelerated multiplication of the Rademacher function matrix for m=5 and the vector

Conclusion

1. When jointly detecting and estimating the parameters of a set of noise-like CNS, including their copies randomly shifted relative to each other in frequency and time, using the maximum likelihood criterion against a background of white Gaussian noise, it is necessary to calculate the composition of the real parts of their DCFs using correlators or matched filters. In this case, the distribution function of the sum of mutual interference and transformed noise will correspond to a Gaussian law, and the estimated parameters of the CNS will be their carrier frequencies, carrier phases, and relative time delays. As a result, this approach can be used for relatively small phase shifts of the CNS carrier frequencies.

2. For significant phase instabilities of the carrier frequencies of the CNS composition, the DCF modulus is usually calculated, resulting in the distribution function of mutual interference and noise at the input of the receiver's decision device corresponding to the Rayleigh-Rice law. 3. When detecting either a single or a set of signal-to-noise ratios (STRs) with unknown carrier frequencies and time delays, it is necessary to repeatedly calculate the convolutions of the reference and received STRs. In digital terms, the convolution calculation operation is reduced to vector-matrix multiplication. When synchronizing using received STRs, it is reduced to multiplying the circulant matrices of the PRS used to generate the STRs by the vector of signal samples at the receiver input.

4. When using binary STRs, fast multiplication of its circulant matrix by a vector can be implemented using the fast Hadamard transform if a linear transformation of the circulant matrices of the used STRs to a Hadamard matrix is possible. The number of elementary arithmetic operations required in this case is $Nlog_2N = Nm$, while direct matrix-vector multiplication would require elementary N^2 arithmetic operations.

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